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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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OCTOBER 1920

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THE
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VOLUME LII OCTOBER 1920 NUMBER 3

COLOR-SENSITIVENESS OF PHOTO-ELECTRIC CELLS

By ELEANOR FRANCES SEILER

ABSTRACT

Color-sensitiveness curves of thirty photo-electric cells, including all the alkali metals and hydrides of Na, K, Rb, and Cs.—As great care was used in mounting the apparatus rigidly, in calibrating the thermo-couple and wave-length scale, and in maintaining a constant source of light, the curves were accurately determined and this enabled the *wave-lengths of maximum sensitiveness* to be located within about 1 $\mu\mu$. It was found that as the atomic weight of the alkali metal increases, the maximum sensitiveness decreases, the resonance peak becomes broader, and λ_{max} shifts toward the red. The author suggests that these changes may be associated with the increase in atomic volume. For glass cells filled with argon at low pressure, λ_{max} was 405, 419, 440, 473, and 539 $\mu\mu$ for Li, Na, K, Rb, and Cs respectively, while for the corresponding hydrides NaH, KH, RbH, and CsH, the values were larger: 427, 456, 481, and 540 $\mu\mu$ respectively. With neon instead of argon, in the case of NaH and KH, λ_{max} was about 20 $\mu\mu$ shorter. And while, in the case of KH, pyrex cells gave the same curve as glass ones, quartz cells showed a longer λ_{max} for both KH and RbH, contrary to what one would expect from differences in absorption. Since the effects of gas and of cell wall were not eliminated and the normal and selective photo-electric effects were not differentiated, the curves are characteristic of the particular cells rather than of the various metals alone; nevertheless it is of interest to note that the products of λ_{max} with (1) resonance potential, (2) ionization potential, and (3) absolute melting temperature of the corresponding metals are each roughly constant. The hydride cells were somewhat more sensitive than the corresponding metal cells.

Fatigue tests of two photo-electric cells. The effect of illuminating a *K cell* for 525 hours was to increase its sensitiveness by about 70 per cent, whereas a *KH cell* remained constant in sensitiveness during 1000 hours' illumination.

Preparation of a lithium photo-electric cell. It was found possible to dissolve lithium in aethylamine if absolutely dry and if a trace of ammonia was present. A uniformly distributed layer of lithium was obtained by evaporating the solution.

Method of preparing pure aethylamine is described.

I. INTRODUCTION

In 1918 T. Shinomiya[1] carried out an experimental determination of the maximum wave-length color-sensitiveness point (λ_{max}) of photo-electric cells of the pure alkali metals sodium, potassium, and rubidium, and of the colloidal modification of the alkali hydrides of these same metals. He found that the λ_{max} was not the same for the pure metal and the hydride, but that a shift occurred, the nature of which was not fully determined.

The object of this investigation was threefold:

1. To obtain more consistent results on the shifting phenomenon and to secure conclusive experimental proof of its direction and amount by the use of many more cells and the extension of the work to include all the alkali metals. No satisfactory theory has been advanced to explain the cause of this shifting of the wave-length of maximum sensitiveness. However, the results of this investigation prove the existence of such a phenomenon, and furthermore they show the shift to be consistently always in the same direction, namely, toward longer wave-lengths when the alkali metals were sensitized by means of hydrogen.

2. To complete the study of the color-sensitiveness for the whole group of alkali metals. When the photo-electric cell is to be used in making photometric measurements, it is necessary to have a complete knowledge of its curve of sensitiveness. In scientific literature it is found that the curves given contain only a few points, which hardly suffice to determine the point of maximum sensitiveness or the general nature of the curves with much accuracy. The ideal photo-electric cell would have the same color-sensitiveness as the human eye.

3. To furnish data for theoretical purposes in the application of the quantum theory to chemistry. No general agreement has yet been reached regarding the relation of the selective to the normal photo-electric effect. Wiedemann[2] found that hydrogen was essential for the selective effect of potassium. The results of this experiment gave a decided selective effect in cells where the potassium was repeatedly distilled in vacuo. According to Millikan

[1] *Astrophysical Journal*, **49**, 303, 1919.

[2] *Berichte der deutschen physikalischen Gesellschaft*, **18**, 333, 1916.

and Souder[1] there is an "essential identity" of the two effects. This investigation was not carried on with the purpose of getting either effect to the elimination of the other. Primary interest was centered in the photometric properties of the cell and not of the alkali metals as such. Apparently there is considerable difference in sensitiveness when the absorption of light through the glass is taken into account and when gas is present in the cells.

II. THE APPARATUS AND METHOD

A. THE CALIBRATIONS

In order to measure the energy of illumination throughout the visible spectrum and the corresponding wave-lengths of light, it was necessary to make four calibrations, about every month during the course of the investigation. We will call these calibrations: (1) thermo-couple, (2) scale-energy, (3) spectrometer, and (4) scale-wave-length.

1. *Thermo-couple.* A thermo-couple, furnished by W. W. Coblentz from the Bureau of Standards, was calibrated by means of the light from a Hefner lamp. Chemically pure amyl acetate was put into the lamp, which was placed exactly one meter from the thermo-couple and in the same horizontal plane with it. Both were placed in a large two-compartment box, which was completely blackened. The partition contained a shutter, by means of which the light could be allowed to fall upon the thermo-couple, and which acted as a partial diaphragm. A hooded chimney was placed above the lamp to take care of the heat from the flame. A small glass window, with a horizontal line on it, was set in the box directly behind the image of the flame, so that the flame could be accurately adjusted to the proper height and observed from the outside. The box being tight and free from air currents, the flame soon came to an equilibrium condition. Two fine wires led from the thermo-couple to an insulating plug in the back of the box. There they were soldered to heavy double silk-wound copper wires, which led through the plug down a heavy-walled wooden tube to a galvanometer. This tube served the purpose of keeping the temperature

[1] *Proceedings of the National Academy of Science*, **2**, 19, 1916.

constant and of preventing air currents. The lead wires in the tube were insulated by passing down the center over strips of hard rubber. The galvanometer was a Leeds and Northrup high sensitivity type, whose figure of merit was 2.7×10^{-9}. It was inclosed in a box about which tin foil was wrapped for earthing.

These precautions made the thermo-galvanometer deflections exceedingly steady, and a reading could be repeated time and time again with not as much as 1 per cent of variation. The mean of twenty readings gave a deflection of 121 mm. The energy radiating from a Hefner lamp, according to K. Ångstrom,[1] is 1089 ergs per sec. per sq. cm at a distance of one meter. Hence a deflection of 1 mm corresponded to a flow of energy of 9 ergs per sec. per sq. cm. The slit of the thermo-couple was a trifle narrower than the slit L (Fig. 2) admitting light to the photo-electric cell, so that the former was always covered by light. The area of L was 0.2842 sq. cm, and hence the energy passing through that slit per mm of deflection of the galvanometer of the thermo-couple was $9 \cdot 0.2842$ or 2.558 ergs.

2. *Scale-energy.* The thermo-couple was placed with its slit directly behind L and held there rigidly. It was connected to the same galvanometer through another tube, which was completely wrapped with tin foil and earthed.

By means of a fine screw R (Fig. 2), the thermo-couple was moved along the millimeter scale S through the entire spectrum, and the mean of three readings taken as the deflection at each point. Then by plotting a curve with scale-readings as abscissae and energy in ergs as ordinates, the energy incident upon every point in the spectrum was exactly determined.

3. *Spectrometer.* Since the Hilger constant-deviation spectrometer was to be used in determining the wave-length incident at every corresponding point on the scale S, it, too, had to be calibrated. This was done by means of a Plücker discharge tube. The prism of the spectrometer was adjusted so that the corrections were very slight.

4. *Scale-wave-length.* The wave-lengths corresponding to the points along the scale were then obtained with this instrument.

[1] W. W. Coblentz, *Bulletin of Bureau of Standards*, **11**, 95, 1914.

The lines seen in the spectrometer were almost as fine as those seen when the discharge tube was used, showing that the light was very nearly monochromatic. They became somewhat broader toward the red end of the spectrum, but this was unavoidable and, as the experiment did not require great accuracy in that region, accuracy was sacrificed to get better definition in the other end of the spectrum. By plotting a curve with wave-lengths as abscissae and scale-readings as ordinates, the wave-length corresponding to any position on the scale was obtained.

B. THE EXPERIMENT

1. *The optical system.* The optical system used in this investigation was an improvement over that used by T. Shinomiya in that, by the use of two large prisms and a condensing lens L_4 (Fig. 1),

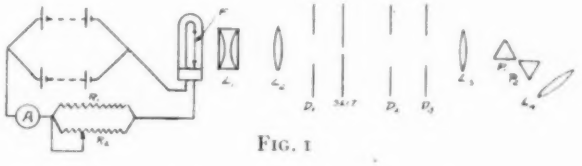

FIG. 1

greater dispersion and better monochromatism were obtained. As the sensitiveness of the photo-electric cell changes in a continuous way along the spectrum, the very slight deviation from the ideal condition of absolute monochromatism had little effect on the final curves.

The set-up is shown diagrammatically in Figure 1. The source of light, F, was a specially constructed tungsten 17-ampere nitrogen-filled lamp made by the Nela Research Laboratory. The bright filament was a straight, narrow flat strip which acted as a line source. A Weston direct-current ammeter, A, measured the current through the lamp. This had to be kept very constant at 17 amperes, for the slightest variation in the intensity of the light made itself felt in both the thermo and photo-electric galvanometers. The current was adjusted by a tin resistance, R_1, and a fine adjustment sliding resistance R_2 in parallel. Two sets of storage batteries connected in parallel served as the source of current. They were kept well charged and the resultant current was steady.

The light was condensed by lenses L_1 and L_2 and brought to a focus on the slit S, which was 0.7 mm in width. It was then rendered parallel by the lens L_3. The diaphragm D_3 was adjusted so that the front face of the prism P_1 was just covered by light. The position of minimum deviation for green-blue light was chosen for both prisms. After passing through the prisms the various

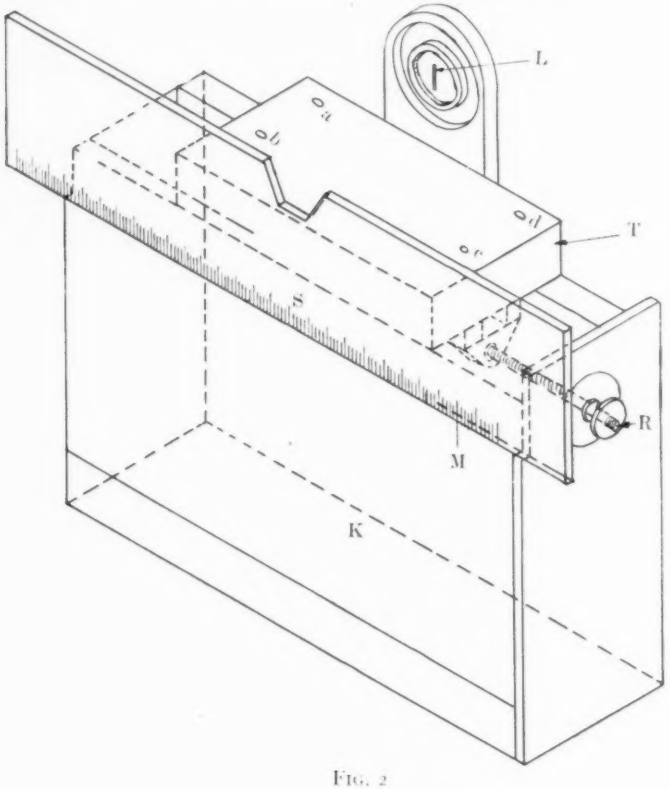

FIG. 2

colors of the spectrum were brought to a focus and made pure by the lens L_4. The spectrum, 10 cm in length, was beautifully distinct and bright. The reflected and stray light was eliminated by the diaphragms D_1, D_2, and D_3, and further by placing the entire optical system in a box made of thin compo-board, completely blackened on the inside. The slit L (Fig. 2) was accurately located

in the focal plane of the lens L_4. It was mounted on the table T, which could be moved through the spectrum by means of the fine screw R. This table was inside a heavy black box K, rigidly attached to a solid pier. The entire optical system was made purposely heavy, so that not even the slightest displacement could take place and hence change the calibrations. One can readily see that a very small change in any part of such a complicated system would throw the whole arrangement out of order. The scale, S, on the outside of the box was rigidly connected with the slit L. It was the sliding of this scale past a fine reference mark M that was calibrated, first in ergs and then in wave-lengths of light.

The mounting for the slit L was made to exactly fit the mounting of the thermo-couple used in the energy calibration. On the top of the table were bored four holes, a, b, c, d (Fig. 2). Into these four holes fit the legs of a light-tight metal box, specially constructed for containing the photo-electric cell under test. The shutter on this box for admitting and cutting off the light to the cells was situated so that it was directly behind the slit L. This shutter was manipulated by the operator at his observing position by means of pulleys and strings. The apparatus was so arranged that at the same place where these strings were situated the operator could adjust the resistance R_2, read the ammeter A, and note the photo-electric galvanometer deflections.

The metal box container had two hard rubber insulators for supporting the photo-electric cell and two insulating plugs and binding posts for the lead wires. The positive lead was made of sulphur, and the negative one of hard rubber. A binding post attached directly to the box served the purpose of grounding the box and the outside of the photo-electric cells.

2. *The photo-electric cells.* There were thirty cells used in the investigation, including all of the alkali metals and the colloidal modification of the alkali hydrides of sodium, potassium, rubidium, and cassium. They all contained argon at a low pressure, except two cells where neon was used. Three kinds of glass—common, pyrex, and quartz—were investigated. All of the cells were made

by Dr. Jakob Kunz in this laboratory. A full description of their construction, with the exception of lithium, has been given.[1]

The making of the lithium cell was somewhat of a task, because of the danger encountered in trying to distil lithium, and of the difficulty in obtaining a solute for it. At the suggestion of Dr. A. G. Loomis, of the chemistry department, we tried dissolving it in aethylamine, and were eventually successful. The necessary condition for its solution is that the aethylamine be absolutely dry and that it be in the presence of a trace of ammonia.

The aethylamine was made according to the following directions, the chemical reaction being

$$(C_2H_5NH_2+HBr)+NaOH=NaBr+C_2H_5NH_2+H_2O.$$

Seventy grams of aethylamine hydrobromide were placed in a 500-cc pyrex glass flask, fitted with a tight cork stopper holding a separatory funnel and a reflux condenser kept at about 30° C. To the top of the reflux condenser was attached a drying column of solid NaOH, and this connected to a downward condenser of the spiral type, cooled with ice and salt. The receiver was also cooled with a mixture of crushed ice and salt and had a NaOH tube attached. All stoppers were of cork to prevent the product from becoming yellow because of attack of rubber by the amine.

About 200 grams of a 30 per cent solution of NaOH was slowly dropped into the flask through the separatory funnel, and when all had been added, the mixture was first heated with warm water and later with a flame, refluxing until no more amine condensed into the receiver. The crude amine was then redistilled three or four times, using metallic lithium as a drying agent. The final product was a pure, colorless, dry fluid. Metallic lithium was put into it, in a vessel connected with a cell. After standing ten hours in the aethylamine in the presence of a trace of ammonia, the lithium was dissolved and the clear liquid became a deep-blue color. This solution was poured into the cell and the aethylamine pumped out, leaving a uniformly distributed surface of grayish blue lithium. The cell was then partly filled with argon at low pressure and sealed off.

[1] *Physical Review*, **7**, 62, 1916.

3. *The set-up.* A diagrammatic sketch of the connections is shown in Figure 3. The source of potential E was a set of new dry cells. The voltage ranged anywhere from 100 to 310 volts, depending upon the kind of cell used. The potential was measured by a Weston direct-current voltmeter, V. All of the connecting wires, as well as the battery itself, were insulated by paraffin.

The galvanometer, G, which measured the photo-electric current, was a Leeds and Northrup high-sensitivity type. It had a sensitiveness of 8×10^{-10} amperes for a scale-distance of 1.25 m, and a critical damping resistance of 9600 ohms. To safeguard against short circuit, a resistance of 100,000 ohms was inserted in the

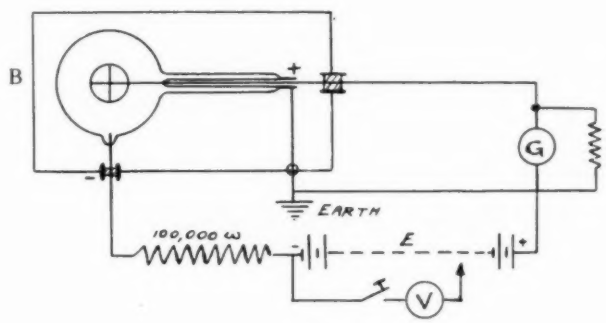

FIG. 3

circuit. The outside of the cell, the positive terminal of the battery, and the box B, figure 3, containing the photo-electric cell, were all earthed. When working with the lithium cell, a quadrant electrometer had to be used to measure the photo-electric current, because of the small light intensities in the region of its maximum sensitivity, and the rather low sensitiveness of the cell.

4. *Method of procedure.* After the calibrations were made, great care was taken that no part of the system was disturbed. Then a characteristic curve was made of the first cell to be tested, after which a preliminary sensitiveness-curve was run and finally the real sensitiveness-curve data obtained. This procedure was repeated for all the cells used. The process will now be discussed a little more in detail.

The cell was mounted in the box K (Fig. 2), the approximate point of maximum galvanometer deflection was found by referring to the work of T. Shinomiya, and the cell placed at that particular wave-length. Starting with a low voltage, around 80, until a deflection of about 40 mm was obtained, the cell was moved back and forth until the exact maximum was found by trial. Keeping this position, the deflections for varying voltages were noted. The upper limit of the voltage was determined by the unsteadiness of the galvanometer deflections. This unsteadiness indicated that the arcing potential was being reached. If the cell was allowed to glow much, its sensitiveness decreased. A slight glowing oftentimes increased the sensitivity. Glowing could easily be detected by the galvanometer deflections suddenly going off the scale. Whenever this happened, an entirely new characteristic had to be run. The Na and NaH cells were the most difficult to handle, for the characteristic curve rises so rapidly that a variation of 4 volts (one dry cell) often caused great unsteadiness in the deflections, and sometimes glowing. Barring this unsteadiness, the other readings were very steady and could readily be repeated at the time when the data were taken. If, however, a characteristic was run shortly after a cell was made, and again a day or two later, it was found to change somewhat. After a week's time very little change took place. All the data were taken on cells more than a week old. This variation was more marked in the hydrides of the alkali metals than in the pure metals.

Immediately after running a characteristic, the curve was drawn, plotting volts against photo-electric current. Some characteristic curves are shown in Shinomiya's paper.

A constant potential difference was then chosen from the horizontal portion of the curve, or at most up only a short distance on the vertical part. In no case was such a voltage used as to make the galvanometer deflections unsteady, which occurs when the critical potential is approached.

Having selected a safe potential, one that would give reasonably large deflections (100 mm or more), the cell was started at the extreme violet end of the spectrum and by steps of a centimeter was taken up through the red end. This was the preliminary

run to determine approximately the position, not of maximum galvanometer deflection, but of maximum photo-electric sensitiveness. This point varied somewhat from the former and was found by dividing the deflections by the corresponding energy at the given scale-reading. After a lapse of about fifteen minutes, during which time the cell was left in the dark, a second final run was made, again starting at the violet end. If no lapse of time is allowed to take place between the first and second sets of data, the later readings will be higher, because for a short time the cell becomes more sensitive when it has been under the influence of light. Two readings, rarely differing, were taken at each point, and this time they were taken only 3 mm apart in the region of maximum sensitivity. Thus the maximum point was accurately determined to within one $\mu\mu$.

Finally curves were plotted with wave-lengths of light expressed in $\mu\mu$ as abscissae and specific photo-electric sensitiveness as ordinates, the units being 0.61×10^{-11} coulombs per erg.

III. EXPERIMENTAL RESULTS

The question as to how the colloidal modification of the alkali hydrides affect the color-sensitiveness has been studied. It was found that the results of sensitizing by hydrogen were twofold:

a) It shifts the maximum sensitiveness, λ_{max}, to longer wave-lengths by an amount depending upon the alkali used. As can be seen from Table I below, where λ_{max} and the applied voltage of all the cells investigated are listed, NaH shifted 8 $\mu\mu$, KH 16 $\mu\mu$, RbH 8 $\mu\mu$, and CsH gave only the slight change in one $\mu\mu$. Figures 6, 7, and 8 show this shifting graphically.

b) The colloidal modification of the alkali hydrides also increases the sensitiveness; that is, a greater photo-electric current is obtained with a given applied voltage from a hydride cell than from the pure metal. The sensitiveness depends upon the applied voltage, but on the whole the results indicate a larger ordinate for the hydride cells. Pohl and Pringsheim[1] suggest that this increased sensitiveness is due to the greater ease with which the electrons

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **13**, 211, 1911; **15**, 179, 1911.

TABLE I

Type of Cell	λ_{max}	Applied Voltage	Type of Cell	λ_{max}	Applied Voltage
Li	405	246	KH Quartz	462	145
Na 1	419	149	KH Pyrex 1	457	154
Na 2	419	116	KH Pyrex 2	457	127
Mean for Na	419		Rb 1	473	146
			Rb 2	473	123
NaH 1	427	304	Rb 3	473	136
NaH 2	426	182	Rb 4	473	111
NaH 3	427	308	Mean for Rb	473	
NaH 4	428	171			
NaH 5	429	140	RbH 1	479	140
Mean for NaH	427		RbH 2	483	101
			RbH 3	481	123
NaH Neon	407	285	Mean for RbH	481	
K 1	440	167			
K 2	440	172	RbH Quartz	507	179
Mean for K	440		Cs 1	540	131
			Cs 2	538	117
KH 1	457	119	Mean for Cs	539	
KH 2	457	136			
KH 3	454	140	CsH 1	539	148
Mean for KH	456		CsH 2	541	135
			Mean for CsH	540	
KH Neon	438	127			

TABLE II

CALIBRATION DATA GIVING THE WAVE-LENGTHS AND ENERGY ALONG THE ARBITRARY SCALE *S*

Scale-Reading	Wave-Length in $\mu\mu$	Energy in Ergs	Scale-Reading	Wave-Length in $\mu\mu$	Energy in Ergs
40.0	393.0	1.53	45.3	456.7	73.8
40.5	397.3	2.04	45.5	462.0	84.4
41.0	401.0	3.84	45.7	466.7	94.3
41.3	403.2	4.59	46.0	473.1	109.1
41.5	404.9	5.89	46.3	483.2	127.3
41.7	406.6	6.55	46.5	488.7	137.9
42.0	409.5	8.68	46.7	496.6	151.5
42.3	412.0	10.48	47.0	506.3	172.0
42.5	414.6	12.56	47.3	516.7	191.9
42.7	416.5	14.67	47.5	524.3	205.7
43.0	419.6	17.35	47.7	535.2	221.0
43.3	423.3	21.22	48.0	550.6	239.0
43.5	425.4	23.58	48.3	567.7	259.2
43.7	428.5	26.85	48.5	580.2	270.1
44.0	433.0	32.70	48.7	593.5	288.2
44.3	438.0	41.30	49.0	616.6	305.3
44.5	440.8	45.20	49.5	660.7	338.5
44.7	445.0	51.75	50.0	718.0	366.7
45.0	451.0	61.80			

can emerge from the tiny globules of metal which characterize the colloidal state.

The curves of color-sensitiveness have been obtained for all the pure alkali metals and for all the hydrides with the exception of Li. The curves resemble resonance curves, and Pohl and Pringsheim have suggested that the selective effect is a molecular resonance phenomenon in which the electron follows the electric force. The curves, however, are not pure resonant ones because of lack of perfect symmetry. If a center line is drawn from the peak of a curve perpendicular to the wave-length axis and then about halfway between the peak and the axis another line is drawn perpendicular to this center line, the distances measured along the latter from the center to the two points where it cuts the curve is a measure of the curve's symmetry. In the pure metal as well as in the hydride of the sodium and potassium cells, the distance from the central line to the right side of the curve was greater than that to the left; Cs and CsH, on the other hand, showed dissymmetry in the opposite direction. The Rb and RbH cells are almost symmetrical. A sufficient number of points are located on each curve so that its exact nature is determined. Many of the points have been omitted from the drawings for the sake of clearness. They all, however, fell upon the smooth curve. An examination of the curves and Table I clearly shows how consistently the cells operated. The position of λ_{max} for individual cells of the same metal fell in almost exactly the same position.

Figures 6 and 7 show how the gas in the cells affects the photo-electric effect. A sodium cell and a potassium cell, each filled with neon instead of the argon of the other cells, showed a decided shift of the λ_{max} toward shorter wave-lengths. The amount of shift was 20 $\mu\mu$ for sodium and 18 $\mu\mu$ for potassium. That there should be such a difference between the λ_{max} for the cells filled with neon and with argon, both being inert gases, is a rather surprising phenomenon. It is probably connected with the fact that the maximum sensitiveness of sodium and potassium in these cells deviates considerably from the values obtained for pure metals without gas. This question requires further study.

Figures 6 and 7 indicate the effect that the kind of glass used in the photo-electric cell has on the wave-length of maximum sensitiveness. Most of the cells were made of common glass, but two cells of pyrex (KH Pyrex 1 and KH Pyrex 2) and two of quartz (KH Quartz and RbH Quartz) were constructed. Both pyrex cells

TABLE III

VARIATION IN ALL ALKALI METALS*

	Li		Na 2		K 2		Rb 3		Cs 2	
SCALE IN CENTIMETERS	Electrical Deflection	Deflection Energy	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness
	mm		mm		mm		mm		mm	
40.0	14.8	1265.0	1.2	62.2						
41.0	43.8	1194.0	4.8	99.8	2.0	41.7	0.7	14.6		
41.5	70.7	1574.0	8.9	120.9	3.4	46.2				
42.0	101.2	1527.0	16.1	148.1	6.6	60.7	2.0	18.4	1.8	16.6
42.3			21.4	163.2						
42.5	133.3	1390.0	26.1	166.2	11.9	75.8				
43.0	166.5	1256.0	36.6	168.5	21.2	97.6	6.1	28.1	3.9	17.9
43.5			48.3	163.6	36.8	124.7				
44.0			57.0	139.2	55.8	136.4	20.2	49.4	9.0	22.0
44.3			60.4	116.9						
44.5	302.5	877.0			78.6	139.0	33.5	59.21		
44.7			56.8	87.8	88.7	137.0				
45.0	321.7	681.5	53.3	68.9	96.7	124.9			21.3	27.51
45.3					107.2	116.1	73.0	79.0		
45.5	307.0	476.8	51.0	47.5	113.3	107.4	86.5	81.9	32.9	31.2
45.7					113.3	90.0	101.3	85.8		
46.0	250.0	299.7	48.7	35.6	111.4	81.6	120.8	88.6	47.4	34.7
46.3					103.7	63.2	140.4	85.0		
46.5			44.8	25.9	93.9	54.4			69.7	40.4
46.7					81.1	42.8	139.7	73.6		
47.0	104.7	79.8	41.5	19.3	57.9	27.0	139.0	64.7	104.7	48.7
47.3					33.2	13.9			123.9	51.6
47.5			36.6	14.2	21.2	8.3	123.3	47.9	145.0	56.4
47.7									166.2	60.1
48.0	21.5	11.8	31.0	10.4	6.9	2.3	59.0	19.7	169.0	56.6
48.3									144.2	44.5
48.5			18.8	5.6	3.4	1.0	17.8	5.2.5	117.2	34.7
48.7									94.7	26.2
49.0	8.0	3.4	10.7	3.1	2.0	0.6	8.4	3.0	48.2	12.6
49.5			7.8	1.8	1.2	0.3	4.3	1.0	11.1	2.6
50.0	4.8	1.7			1.1	0.2	2.1	0.5	3.8	0.9

* The sensitiveness column in each case but Li is multiplied by 10^{-11}, giving the specific photo-electric sensitiveness in coulombs per erg.

gave excellent curves, whose λ_{max} varied only one $\mu\mu$ from that of KH cells made from common glass. The two cells constructed of quartz, however, showed a definite shift in λ_{max} toward longer wave-lengths, by an amount equal to 6 $\mu\mu$ for the KH quartz and 26 $\mu\mu$ for the RbH quartz cell. If this shift due to the quartz was caused by absorption of light in the wall of the cell, one should expect it to be in the opposite direction.

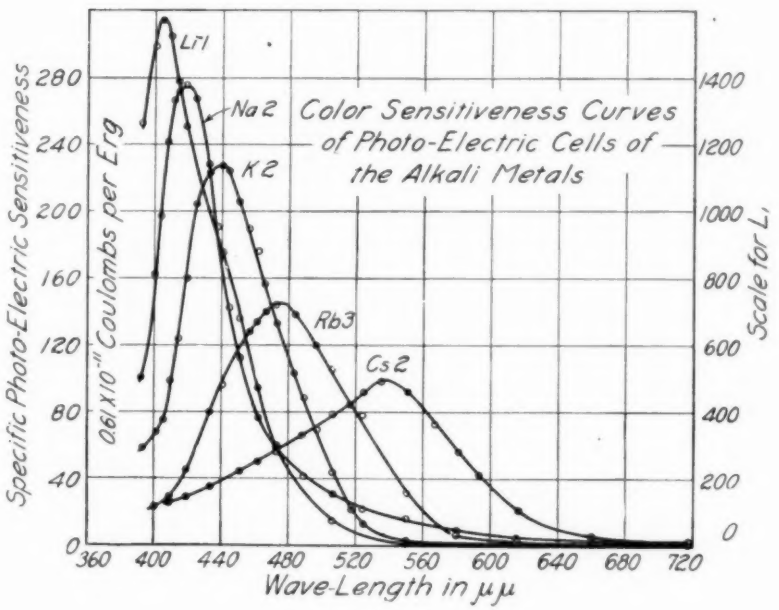
Color Sensitiveness Curves
of Photo-Electric Cells of
the Alkali Metals
Li1
Na2
K2
Rb3
Cs2
Specific Photo-Electric Sensitiveness
.061 X 10⁻¹⁴ Coulombs per Erg
Scale for Li
Wave-Length in μμ
0
40
80
120
160
200
240
280
0
200
400
600
800
1000
1200
1400
360
400
440
480
520
560
600
640
680
720

FIG. 4a

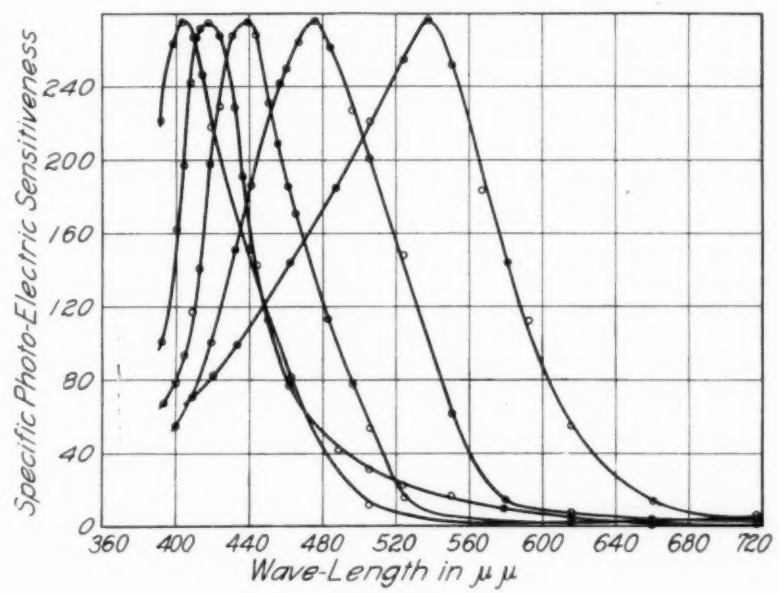
Specific Photo-Electric Sensitiveness
Wave-Length in μμ
0
40
80
120
160
200
240
360
400
440
480
520
560
600
640
680
720

FIG. 4b

A further experiment was carried on for the purpose of detecting a fatigue effect in a K and a KH cell. The temperature, intensity of light, and applied voltage were kept constant and the galvanometer deflections observed at intervals of from ten to twenty-four hours. The voltage continuously applied to the K cell was 110 and after

TABLE IV

VARIATIONS IN THE HYDRIDE CELLS*

SCALE IN CENTIMETERS	NaH 1		KH 1		RbH 2		CsH 2	
	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness
	mm		mm		mm		mm	
40.0	5.8	302.2						
41.0	15.9	331.1	2.5	52.0	1.1	22.9		
41.5	26.8	304.2	3.9	53.0				
41.7	32.3	394.0						
42.0			6.4	58.7	2.8	25.7	1.1	10.1
42.3	59.0	449.5						
42.5			10.5	70.0				
43.0	108.3	500.2	17.5	80.6	7.3	33.0	2.3	10.6
43.5	152.5	516.6	30.0	101.7				
44.0	203.0	496.0	50.0	122.0	23.5	57.4	11.9	12.0
44.5	195.5	345.9	81.2	143.7	38.7	68.4		
44.7			98.7	152.4				
45.0	183.0	236.4	127.3	164.5	62.9	81.2	13.0	16.8
45.3			155.7	168.6	81.9	88.6		
45.5	161.6	153.2	172.4	163.4	97.0	91.9	21.0	19.9
45.7					113.5	96.2		
46.0	134.5	98.3	190.2	139.5	137.5	100.8	35.3	25.9
46.3					164.0	102.9		
46.5	107.0	62.0	183.6	107.6	170.6	99.0	57.3	33.2
46.7					174.3	91.9		
47.0	82.2	38.1	161.9	75.2	171.0	79.5	95.3	44.3
47.3							121.5	50.8
47.5	54.5	21.2	120.4	49.1	138.2	53.7	147.1	57.2
47.7							172.4	62.3
48.0	30.3	10.1	85.3	28.5	88.9	29.7	176.4	58.9
48.3							151.7	40.8
48.5	14.0	4.1.5	45.8	13.6	40.3	13.7	106.5	31.4
48.7							77.7	21.5
49.0	7.0	1.9	20.1	5.3	20.1	5.3	31.6	8.3
49.5	4.6	2.0	10.0	2.4	7.7	1.8	5.6	1.3
50.0	3.3	0.7	7.0	1.5	3.3	0.7	1.7	0.4

*The sensitiveness column in each case is multiplied by 10^{-11}, giving the specific photo-electric sensitiveness in coulombs per erg.

five hundred and twenty-five hours' exposure to strong light the cell had *increased* in sensitiveness by quite a large amount. The galvanometer deflections began with 176 mm and gradually increased up to 300 mm at the end of the exposure. The KH cell, on the other hand, under the same conditions, except that the applied voltage was 36, showed an exceptional constancy. Slight

fluctuations occurred during the one thousand hours' exposure; the initial and final galvanometer deflections were 194 mm and 191 mm, respectively, showing that the fatigue is negligible.

These same two cells were exposed for 60 hours *without* continuously applying a voltage. At the times when the photo-electric galvanometer deflections were taken, a voltage of 100 was applied to the potassium cell and of 27 to the potassium hydride cell. All the conditions remained constant throughout the experi-

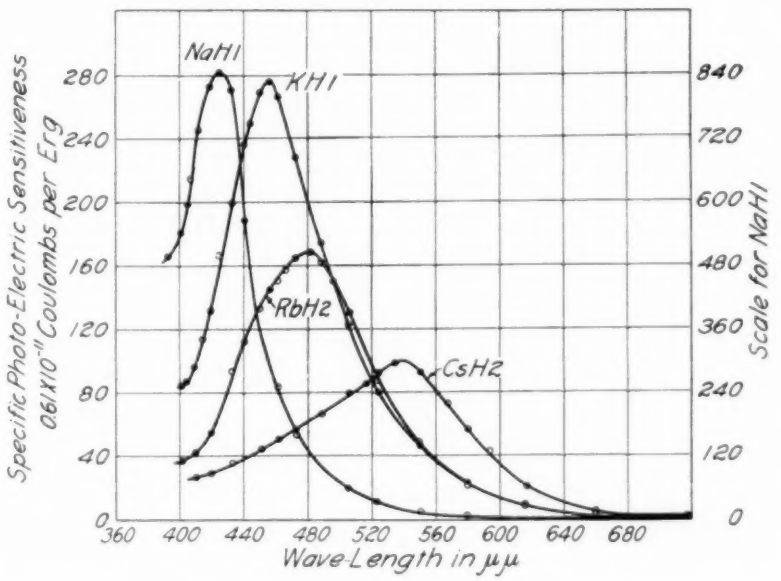

FIG. 5

ment. The results showed an *increase* in sensitiveness for both cells, the initial and final deflections for the potassium cell being 206 mm and 274 mm, and those for the potassium hydride cell being 196 mm and 316 mm. It seems strange that merely the exposure to light, without the passage of a photo-electric current, should increase the sensitiveness so much more rapidly than when the applied voltage caused a current to flow constantly through the cells. Both investigations, however, point to a negative conclusion in regard to a fatigue effect.

TABLE V

VARIATIONS IN POTASSIUM CELLS*

SCALE IN CENTIMETERS	K 1		KH 2		KH QUARTZ		KH PYREX 1		KH NEON	
	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness
	mm		mm		mm		mm		mm	
41.0	3.8	79.0	2.5	52.0	1.0	20.8	1.7	34.7	5.8	120.7
41.5	0.2	84.2	.....	.....	1.6	21.8	.....	.....	9.6	130.4
42.0	9.8	90.2	6.0	55.2	2.6	23.9	9.6	42.2	16.9	148.1
42.5	15.2	96.7	10.0	63.6	4.2	26.7	9.0	57.4	27.2	173.0
43.0	23.5	108.2	16.5	76.0	6.9	31.8	15.1	69.4	44.0	202.6
43.5	36.1	122.4	28.4	96.2	11.8	39.9	32.2	109.3	70.7	239.6
44.0	53.0	129.4	49.0	119.6	20.1	49.0	53.2	130.2	105.0	256.5
44.5	74.3	131.4	80.2	141.8	33.8	59.8	.....	.....	145.3	256.9
44.7	84.1	129.8	98.7	152.4	42.7	66.0	96.6	149.0	161.8	249.8
45.0	98.0	127.4	131.3	169.8	59.8	77.3	122.7	158.5	187.0	241.9
45.3	.....	.....	162.3	175.5	79.4	85.8	154.4	167.0	207.5	224.3
45.5	124.8	118.3	178.0	168.6	92.0	87.2	171.5	162.5	213.0	201.8
45.7	129.5	109.7	.....	.....	102.5	86.8	.....	.....	.....	.....
46.0	128.9	94.4	194.8	142.6	112.8	82.6	200.0	146.4	204.0	149.5
46.3	119.7	75.2	.....	.....	117.0	73.4	.....	.....	.....	.....
46.5	108.5	62.9	177.9	103.0	117.7	68.2	210.5	122.0	170.0	98.5
46.7	93.9	49.5	.....	.....	119.3	62.9	.....	.....	.....	.....
47.0	66.8	31.1	144.8	67.4	120.0	55.8	212.0	98.4	118.0	54.8
47.3	41.0	17.1	.....	.....	116.9	48.7	.....	.....	.....	.....
47.5	27.3	10.6	90.6	32.2	110.7	43.0	184.0	71.5	65.1	25.3
48.0	10.4	3.5	38.3	12.8	72.0	24.0	126.1	42.1	30.0	10.0
48.5	6.7	1.9	14.0	4.2	26.3	7.7	56.2	16.6	14.8	4.4
49.0	5.0	1.3	7.3	1.9	6.1	1.6	20.0	5.2	7.5	2.0
49.5	3.9	0.9	5.2	1.2	1.7	0.4	11.5	2.7	3.5	0.9
50.0	3.0	0.7	4.0	0.9	0.8	0.2	9.4	2.1	1.9	0.4

* The sensitiveness column in each case is multiplied by 10^{-11}, giving the specific photo-electric sensitiveness in coulombs per erg.

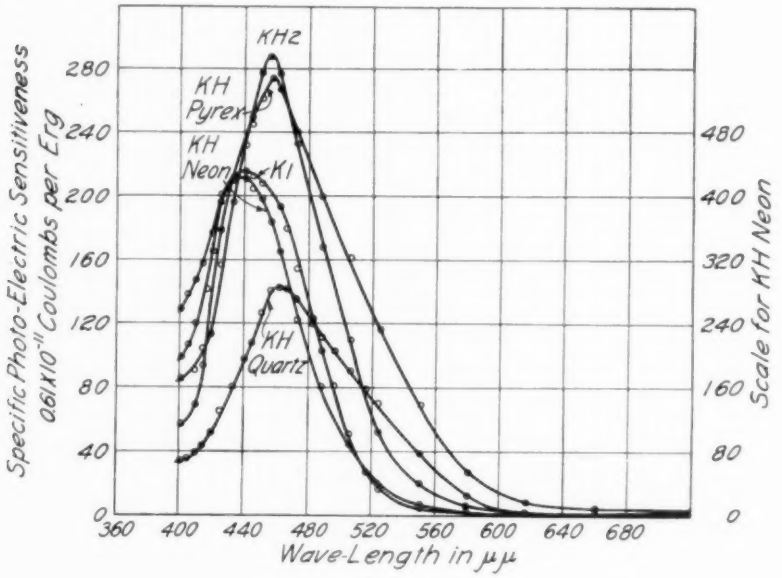

FIG. 6

TABLE VI

VARIATIONS IN SODIUM CELLS*

Scale in Centimeters	Na 1		NaH 4		NaH 5		NaH Neon	
	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness
	mm		mm		mm		mm	
40.0	1.2	.....	1.5	78.2	0.9	46.5	2.3	119.9
41.0	8.2	170.5	4.0	83.2	2.9	60.4	6.4	133.3
41.5	16.8	228.3	6.5	88.3	4.9	66.6	10.2	138.7
42.0	29.0	267.1	.....	.....	8.2	75.4	15.2	139.0
42.3	.....	.....	13.4	102.2	.....	.....	17.8	135.7
42.5	46.0	292.0	.....	.....	13.2	84.0	.....	.....
43.0	.....	.....	24.9	114.5	20.1	92.6	27.2	125.3
43.3	.....	.....	31.5	118.5	25.6	96.4	31.5	118.5
43.5	83.8	284.0	36.8	124.7	29.9	101.4	.....	.....
43.7	.....	.....	42.0	124.9	35.0	104.2	.....	.....
44.0	106.0	258.9	49.9	121.8	.....	.....	41.2	100.5
44.3	.....	.....	.....	.....	.....	.....	.....	.....
44.5	125.9	222.4	62.8	111.1	57.1	100.8	48.8	86.2
44.7	134.0	207.1	.....	.....	64.9	96.7	50.0	77.2
45.0	146.9	189.9	76.3	98.6	.....	.....	53.9	69.7
45.3	157.2	170.1	.....	.....	.....	.....	.....	.....
45.5	162.8	154.2	91.2	86.4	97.2	81.2	.....	.....
46.0	165.0	120.8	100.0	73.4	.....	.....	58.0	42.5
46.5	148.1	85.8	96.0	55.6	96.0	55.5	.....	.....
47	107.3	49.8	71.8	33.4	79.8	37.1	45.2	21.0
47.5	.....	.....	34.3	13.4	.....	.....	29.9	11.7
48.0	18.2	6.1	11.3	37.8	21.0	.....	13.2	4.4
48.5	9.2	2.8	.....	.....	.....	.....	.....	.....
49.0	6.4	1.7	2.6	0.7	0.8	0.2	1.1	0.3
49.5	4.5	1.0	.....	.....	.....	.....	.....	.....
50.0	3.0	0.7	1.5	0.3	.....	.....	0.3	0.1

* The sensitiveness column in each case is multiplied by 10^{-11}, giving the specific photo-electric sensitiveness in coulombs per erg.

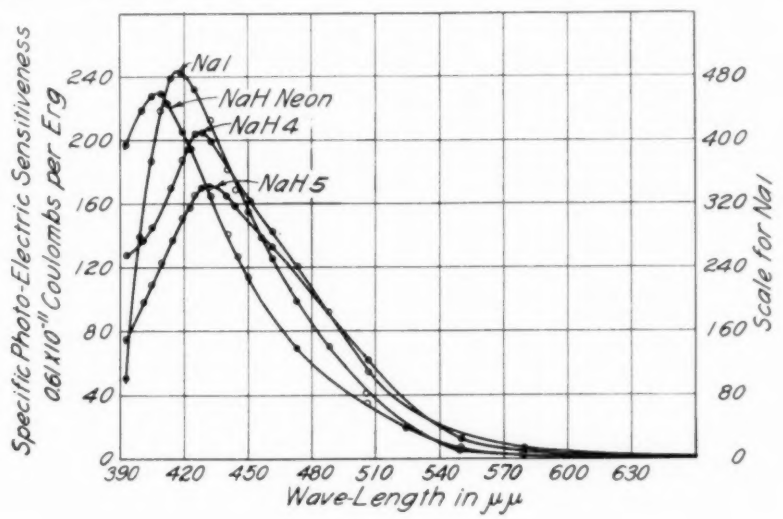

FIG. 7

TABLE VII

VARIATION IN RUBIDIUM AND CAESIUM CELLS*

SCALE IN CENTIMETERS	Rb 2		RbH 3		RbH Q		Cs 1		Cs H 1	
	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness	Galvanometer Deflections	Specific Photo-Electric Sensitiveness
	mm		mm		mm		mm		mm	
41.0	1.0	20.8			0.5	10.7				
41.5										
42.0	2.8	25.7	5.3	30.4	1.2	11.0	0.7	6.3	1.0	9.2
42.3										
42.5										
42.7										
43.0	7.8	35.9	7.2	33.1	2.6	12.0	1.5	6.9	2.1	9.7
43.3										
43.5										
43.7										
44.0	22.3	54.5	18.8	45.9	6.1	14.9	3.4	8.3	6.9	16.8
44.3										
44.5	35.0	61.9	29.9	52.3	10.5	18.6				
44.7										
45.0	55.5	71.8	50.0	64.6	18.3	23.7	8.6	11.1	20.1	26.0
45.3	72.8	78.8	67.2	72.8	25.5	27.6				
45.5	85.5	81.0	80.9	76.6	31.8	30.1	14.7	13.9	31.6	30.0
45.7	99.6	84.4	97.9	83.0	41.4	35.8				
46.0	119.0	87.2	122.1	89.5	51.2	41.2	24.8	18.2	48.7	35.7
46.3	134.5	84.4	145.5	91.3	77.2	48.5				
46.5	138.3	80.1	155.9	90.4	91.6	53.1	41.0	23.7	71.6	41.5
46.7	138.6	74.2	162.0	85.4	110.6	58.3	51.2	27.0	83.1	44.4
47.0	126.6	58.8	157.9	73.4	131.8	61.2	69.8	32.5	106.0	49.3
47.3					144.0	59.9	90.0	37.5	120.5	52.6
47.5	84.4	32.8	117.0	45.5	140.2	50.8	108.7	42.2	150.4	59.0
47.7					147.7	53.4	126.2	45.6	171.5	62.0
48.0	40.5	13.6	63.4	21.2	129.3	43.3	132.1	44.2	171.0	57.2
48.3							122.5	37.8	147.6	45.5
48.5	17.0	5.0	27.7	8.2	87.1	25.7	93.8	27.7	103.9	30.7
48.7									75.8	21.1
49.0	6.1	1.6	12.4	3.2	48.5	12.7	30.5	95.8	29.0	7.7
49.5	2.1	0.5	6.0	1.4	25.9	6.1	6.8	1.6	4.3	1.0
50.0	0.9	0.2	4.1	0.9	8.9	2.3	2.8	0.6	1.3	0.3

*The sensitiveness column in each case is multiplied by 10^{-10}, giving the specific photo-electric sensitiveness in coulombs per erg.

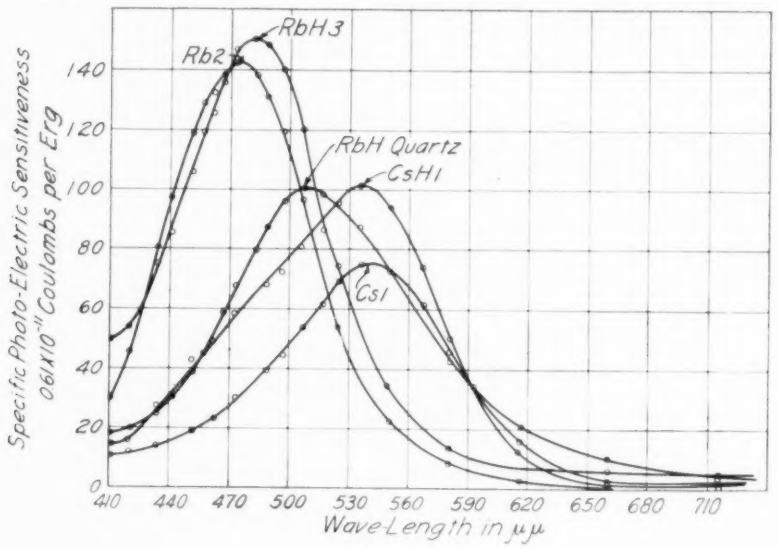

FIG. 8

IV. THEORETICAL CONSIDERATIONS

The results here obtained cannot be used directly for theoretical considerations, because of the effect which absorption of light in the glass wall and the presence of gas in the cell have upon them. Moreover, these results are due to both the normal and the selective effects, for they have not been separated in the present study. Conclusions which have been drawn from previous results are only approximately true. Certain equations to follow, are deduced from theoretical considerations, and when the values from this study are substituted in them, the results show a coincidence of about the same degree of approximation as those previously obtained.

Since the curves resemble resonance curves, it is natural to assume that the frequency $n_{ph} = \frac{1}{\lambda_{max}}$, at which the maximum sensitiveness takes place, is connected with the resonance potential and the resonance frequency of the alkali vapors, where the quantum relation $V_r e = hn_r$ and $V_i e = hn_i$ holds for resonance and ionization potentials respectively. All photo-electric phenomena seem to be governed by the quantum relation, so that one might expect proportionality between hn_{ph} and hn_r or hn_i. Thus proportionality may exist between n_{ph} and the resonance and ionization potentials, i.e., the ratios $\frac{n_{ph}}{V_i}$ and $\frac{n_{ph}}{V_r}$ may be constants. These values are calculated both from the λ_{max} found in Pohl and Pringsheim[1] and from this study, and are given in Table VIII.

A glance will show that the latter values are more nearly constant, in spite of the fact that the values of λ_{max} for Na and Li are greatly at variance with the former. In this work they were found to be in the visible spectrum and others have located them in the ultra-violet region. The ionization and resonance potentials are those determined by Tate and Foote for alkali vapors. The greatest variation from constancy in these ratios is in the case of $\frac{n_{ph}}{V_r}$ for Na. It seems probable that the value of V_r for Na is a little high. Why should it be higher than that for Li, for V_i and V_r both seem to decrease as the atomic weight increases with this one exception?

[1] *Die lichtelektrischen Erscheinungen*, p. 33, Table IX.

It was thought that possibly the product of λ_{max} and the melting-point temperature T_m of the alkali metals (Wien's law) might be constant. This calculation shown in Table IX gives fairly good values for the constant C.

The results as graphed in Figures 4 and 5 show clearly that as the atomic weight of the alkali metal decreases, (*a*) the position of λ_{max} shifts toward the violet, (*b*) the sensitiveness in general

TABLE VIII

Metal	Ionization Potential V_i	Resonance Potential V_r	λ_{max}	Frequency n_{ph}	$\frac{n_{ph}}{V_i}$	$\frac{n_{ph}}{V_r}$
Li	5.36	1.84	280	10.68	1.99	5.80
Na	5.11	2.09	340	8.81	1.72	4.21
K	4.32	1.60	435	6.88	1.59	4.30
Rb	4.16	1.55	480	6.23	1.50	4.02
	VALUES OF $\frac{n_{ph}}{V_i}$ AND $\frac{n_{ph}}{V_r}$ FROM THIS INVESTIGATION					
Li	5.36	1.84	405	7.41	1.38	4.03
Na	5.11	2.09	419	7.16	1.40	3.43
K	4.32	1.60	440	6.82	1.58	4.26
Rb	4.16	1.55	473	6.34	1.52	4.09
Cs	3.88	1.45	539	5.56	1.43	3.83

TABLE IX

Element	λ_{max}	T_m	$C \times 10^3$
Li	405	458.0	1.86
Na	419	370.5	1.55
K	440	335.3	1.48
Rb	473	311.0	1.47
Cs	539	299.0	1.61

increases, (*c*) the resonant point, if such it may be called, becomes more marked, that is, the lithium curve is narrower than that of Cs. In order to show this latter point, an extra Figure 4*b* has been made, where the curves are so drawn that the maximum ordinates are all equal.

In regard to the shifting of λ_{max} as we go from Cs to Li, it is known that a similar relation holds for the "convergence frequency" of the principal spectral series for these alkali metals.

It may be connected with the increase in atomic volume. The colloidal state increased the volume and shifted λ_{max} toward longer wave-lengths, so it is natural to suppose that a large atomic volume means that the electron is held less firmly and consequently can be set in motion by a quantum of relatively small magnitude—that is, at a longer wave-length—than in the case of a substance with small atomic volume. Therefore Cs, having an atomic volume five times larger than Li, would have a λ_{max} of a much longer wave-length.

Two formulas, 1 and 2, have been suggested by Lindemann[1] which are partly of an empirical character.

$$n_{ph} = \frac{e}{2\pi}\sqrt{\frac{n_v}{mr^3}} \quad \text{or}$$

$$n_{ph} = \frac{2}{\pi}\frac{e}{}\sqrt{\frac{dn_v}{mM}} \quad . \quad . \quad . \quad . \qquad (1)$$

where n_{ph} is the characteristic violet frequency or the maximum color-sensitiveness frequency, e the charge on the electron in electrostatic units, n_v the valency, m the mass of the electron, r one-half the distance between the centers of two neighboring

TABLE X

Element	d	Atomic Weight	$M \times 10^{-24}$ grams	$n_{ph} \times 10^{-13}$	λ_{max} Calculated	λ_{max} Observed
Li	0.51	6.94	11.17	1.276	235	405
Na	0.97	23.00	37.09	0.991	303	419
K	0.86	39.10	63.02	0.771	390	440
Rb	1.52	85.45	137.60	0.642	467	473
Cs	1.88	132.81	214.20	0.574	523	539

atoms, d the density, and M the atomic mass. Table X shows the comparison between the observed λ_{max} and those calculated from Formula 1.

The second formula is a combination of Haber's unexplained equation $n_{ph} = n_r\sqrt{M/m}$, where n_r is the characteristic red frequency

[1] *Berichte deutschen physikalischen Gesellschaft*, **13**, 1107, 1911.

emitted by the vibrating atom, and Lindemann's melting-point equation:

$$n_r = \frac{1}{2\pi\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}} \sqrt{\frac{3k}{MV^{\frac{1}{3}}}} \sqrt{T_m}.$$

Substituting the latter in the former, we get

$$n_{ph} = \frac{K}{V^{\frac{1}{3}}} \sqrt{\frac{1}{M}} \sqrt{T_m} \qquad (2)$$

where K is a constant and V is atomic volume. A calculation of n_{ph} from Formula 2 shows that the agreement between theory and experiment is unsatisfactory.

V. SUMMARY AND CONCLUSIONS

Experimental results show:

1. That due to the formation of a hydride there is a shifting of the point of maximum color-sensitiveness (λ_{max}) toward longer wave-lengths for all the alkali metals.

2. That due to an increase in the atomic volume λ_{max} shifts toward longer wave-lengths as we go from Li to Cs.

3. That the curves of color-sensitiveness become broader with increase in atomic weight.

4. The curves of color-sensitiveness for the entire alkali group have been completed.

5. That the effect of making photo-electric cells of quartz is to cause the position of λ_{max} to shift toward longer wave-lengths. This is contrary to what one would expect.

6. That the presence of neon instead of argon in the cells of Na and K shifts λ_{max} toward shorter wave-lengths.

7. That no fatigue effect can be detected.

Theoretical results show:

1. The approximate constancy of the product of the wave-length of maximum sensitiveness and the ionization potential for both the pure metals and the hydrides.

2. The approximate constancy of the product of the wave-length of maximum sensitiveness and the melting temperatures.

3. A fair agreement between observed values of λ_{max} and those calculated from Formula 1 of Lindemann.

4. That the agreement between theory and experiment in Formula 2 of Haber and Lindemann is unsatisfactory.

The writer desires to express her appreciation to Professor A. P. Carman for the use of the laboratory facilities, and to Dr. Jakob Kunz for his continued interest and suggestions.

UNIVERSITY OF ILLINOIS
URBANA
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A PHOTOMETRIC STUDY OF U CEPHEI

By RAYMOND SMITH DUGAN

ABSTRACT

Eclipsing variable U Cephei.—The results of a *photometric study* extending from April 1914 to April 1917, during which 14,112 measurements were made, are collected in Table I. The range of variation is $2^{M}.28$ in primary minimum and $0^{M}.09$ in secondary minimum and the probable error of each normal about $0^{M}.018$. With the exception of variation due to eclipse, the *light-curve* is well represented by the equation: $L = 0.958 + .008 \sin\theta - .021 \cos\theta + .029 (\sin 2\theta - 42^{\circ}.6)$. The terms after the first are interpreted to indicate: (1) the greater brightness of the advancing side of the bright star; (2) the reflection effect on the darker star; and (3) a tidal lag, at least in the case of the bright star, of about 24°. This last conclusion, combined with the fact discovered by the author that according to the available evidence the period is nine seconds longer than it was sixty years ago, suggests a *tidal evolution* of great interest. The observations also indicate that the star disks are *two or three times as bright at the center as at the limb.* The *elements of the orbits and other data* are collected at the end of the paper.

B.D.+81°.27 and 81°.30 were found to be *variables.*

The details of observation and discussion, of which this paper contains the principal results, may be found in the fifth number of the *Contribution from the Princeton University Observatory*, which is now in press.

The measures, numbering 14,112, made with the polarizing photometer, with the usual precautions against systematic errors, during the interval April 1914 to April 1917, were combined into normals and discussed along lines readily deduced for the individual peculiarities of the curve from the methods now in general use.

The probable error of a set of sixteen measures of full weight, derived from the deviations of the observations from the computed curve, is $\pm 0^{M}.041$, comparing favorably, when consideration is given to the minor oscillations in the observed curve of which no explanation has been attempted, with the probable errors affecting similar series of observations previously discussed by the writer.

The stars B.D.+81°.27 and 81°.30, otherwise well suited for comparison, were found to be variables, the character of variation of the former being especially troublesome. It is recommended, therefore, that in future some other star be selected for comparison.

TABLE I

TABLE OF NORMALS OF OBSERVATION

Number	Phase	Wt.	$c-v$	$O-C$	Number	Phase	Wt.	$c-v$	$O-C$
1.....	-9^h51^m30	3	$+1^M008$	$+0^M022$	61.....	$+4^h47^m10$	4	$+0^M999$	$+0^M006$
2.....	36.70	5	0.984	+ 1	62.....	4 57.33	3	1.045	+ 39
3.....	27.75	4	0.951	− 31	63.....	5 4.50	5	1.016	+ 5
4.....	9 15.30	4	0.974	− 7	64.....	11.20	5	1.000	− 15
5.....	8 57.90	4	0.991	+ 12	65.....	22.30	4	1.017	+ 2
6.....	45.00	4	0.986	+ 8	66.....	37.30	5	1.027	+ 10
7.....	8 23.10	4	0.966	− 8	67.....	46.30	5	1.029	+ 10
8.....	7 59.10	3	0.995	+ 24	68.....	5 55.40	5	1.006	− 14
9.....	7 14.70	3	0.931	− 36	69.....	6 6.60	4	1.010	− 10
10.....	6 59.20	5	0.956	− 10	70.....	16.40	5	1.018	− 3
11.....	49.70	5	0.942	− 24	71.....	35.40	4	1.063	+ 41
12.....	38.70	4	0.984	+ 19	72.....	6 49.00	4	1.055	+ 32
13.....	26.80	5	0.902	+ 28	73.....	7 5.00	3	1.056	+ 31
14.....	14.80	5	0.961	− 2	74.....	7 46.13	2	1.014	− 14
15.....	6 3.50	5	0.957	− 6	75.....	10 34.43	1	1.006	− 25
16.....	5 56.90	5	0.957	− 5	76.....	11 4.20	3	1.039	− 9
17.....	48.50	5	0.956	− 6	77.....	27.80	3	1.037	− 12
18.....	42.50	5	0.955	− 6	78.....	11 44.70	3	1.050	0
19.....	34.00	5	0.955	− 6	79.....	12 10.70	3	1.044	− 6
20.....	24.30	4	0.914	− 47	80.....	33.00	3	1.063	+ 12
21.....	12.60	4	0.948	− 12	81.....	12 53.90	3	1.109	+ 57
22.....	5 4.13	4	0.933	− 27	82.....	13 34.50	2	1.112	+ 59
23.....	4 44.50	3	0.883	− 57	83.....	14 8.90	3	1.094	+ 43
24.....	3 58.10	2	0.780	− 49	84.....	39.00	4	1.077	+ 27
25.....	3 19.43	3	0.574	− 51	85.....	49.94	4	1.026	− 23
26.....	2 59.38	3	0.429	− 31	86.....	14 59.39	4	1.027	− 22
27.....	44.22	4	0.298	− 19	87.....	15 8.70	5	1.044	− 3
28.....	28.36	3	+0.104	− 46	88.....	16.50	5	1.022	− 24
29.....	2 12.43	2	−0.044	+ 6	89.....	24.80	4	0.996	− 49
30.....	1 40.29	2	0.647	− 32	90.....	35.00	5	1.023	− 21
31.....	22.94	3	0.964	+ 1	91.....	15 46.50	4	1.013	− 31
32.....	1 9.20	4	1.216	− 36	92.....	16 6.80	3	1.013	− 30
33.....	0 56.43	3	1.304	− 4	93.....	22.40	3	1.015	− 27
34.....	45.44	3	1.287	+ 12	94.....	16 34.83	2	1.043	+ 2
35.....	36.22	4	1.306	− 8	95.....	17 21.80	2	1.033	− 5
36.....	21.50	4	1.293	+ 4	96.....	18 2.80	3	1.078	+ 46
37.....	10.06	4	1.291	+ 5	97.....	18 47.00	2	1.107	+ 78
38.....	−0 0.29	3	1.344	− 49	98.....	21 45.60	2	0.983	− 37
39.....	+0 7.93	3	1.325	− 32	99.....	22 17.20	2	1.038	+ 19
40.....	15.50	3	1.288	+ 4	100....	23 6.63	2	0.983	− 32
41.....	23.50	3	1.326	− 35	101....	24.44	3	1.030	+ 16
42.....	33.01	4	1.277	+ 13	102....	40.38	3	1.005	− 8
43.....	45.75	3	1.249	+ 41	103....	23 51.50	4	1.012	0
44.....	0 55.17	5	1.281	+ 9	104....	24 13.63	4	1.007	− 5
45.....	1 2.63	3	1.200	+ 60	105....	26.00	4	1.044	+ 33
46.....	14.72	4	1.137	− 27	106....	35.83	4	1.019	+ 8
47.....	26.28	4	0.928	− 28	107....	46.00	3	1.036	+ 26
48.....	39.28	4	0.618	+ 2	108....	24 57.71	2	1.075	+ 66
49.....	1 57.30	3	0.261	+ 21	109....	25 11.71	3	1.026	+ 18
50.....	2 14.00	5	−0.023	+ 14	110....	18.93	3	1.035	+ 27
51.....	25.50	5	+λ0.126	+ 14	111....	28.64	3	1.052	+ 46
52.....	35.80	5	0.200	+ 25	112....	25 45.60	3	1.002	− 1
53.....	44.40	5	0.346	+ 14	113....	26 2.10	4	1.017	+ 17
54.....	2 55.80	5	0.485	+ 33	114....	15.60	5	1.020	+ 24
55.....	3 8.28	4	0.539	− 15	115....	24.50	5	0.976	− 17
56.....	13.44	4	0.684	+ 91	116....	31.90	5	0.974	− 17
57.....	3 41.30	3	0.833	+ 63	117....	41.60	5	0.970	− 19
58.....	4 2.50	4	0.884	+ 17	118....	26 57.30	4	0.956	− 27
59.....	17.94	4	0.937	+ 17	119....	27 12.10	5	0.968	− 11
60.....	+4 33.70	4	+0.972	+0.007	120....	+27 22.10	4	+ 0.959	−0 014

TABLE I—*Continued*

Number	Phase	Wt.	$c-v$	$O-C$	Number	Phase	Wt.	$c-v$	$O-C$
121.....	+27h32m39	4	+0M989	+0M019	151.....	+35h50m10	4	1M042	−0M005
122.....	40.50	4	0.970	+ .3	152.....	36 0.90	4	1.071	+ 23
123.....	27 50.00	4	0.960	− 1	153.....	36 24.90	2	1.041	− 8
124.....	28 1.90	5	0.973	+ 16	154.....	37 0.25	2	1.054	+ 4
125.....	10.00	5	0.972	+ 19	155.....	38 29.70	4	1.027	− 20
126.....	20.90	4	0.970	+ 21	156.....	42.80	4	1.004	− 49
127.....	34.70	5	0.961	+ 17	157.....	38 56.90	3	1.011	− 42
128.....	42.70	4	0.930	− 11	158.....	39 13.90	4	1.005	− 48
129.....	48.40	5	0.930	− 9	159.....	30.20	4	1.020	− 33
130.....	28 54.70	5	0.945	+ 7	160.....	39 50.00	2	1.024	− 29
131.....	29 0.10	5	0.969	+ 32	161.....	40 8.50	1	1.035	− 17
132.....	8.70	5	0.978	+ 44	162.....	40 19.25	2	1.070	+ 18
133.....	17.00	5	0.941	+ 8	163.....	41 7.30	3	1.069	+ 18
134.....	23.50	5	0.939	+ 6	164.....	24.80	3	1.118	+ 68
135.....	29.25	4	0.966	+ 33	165.....	41 57.42	3	1.053	+ 5
136.....	35.00	5	0.943	+ 10	166.....	42 43.25	3	1.023	− 20
137.....	45.00	5	0.893	− 40	167.....	43 55.17	3	1.059	+ 20
138.....	29 58.00	4	0.933	− 1	168.....	45 48.30	2	1.005	− 19
139.....	30 7.80	4	0.920	− 15	169.....	46 20.20	2	1.068	+ 49
140.....	19.40	3	0.928	− 9	170.....	46 48.10	3	1.043	+ 29
141.....	30 40.42	3	0.959	+ 18	171.....	47 9.50	3	1.050	+ 45
142.....	31 0.70	3	0.922	− 27	172.....	31.10	3	1.045	+ 36
143.....	31 35.58	2	0.973	+ 6	173.....	47 46.30	4	1.055	+ 48
144.....	33 26.50	3	0.999	− 21	174.....	48 12.10	3	0.999	− 2
145.....	33 53.50	3	1.073	+ 43	175.....	48 41.21	2	1.002	+ 5
146.....	34 16.70	4	0.996	− 43	176.....	49 6.80	1	1.004	+ 12
147.....	30.30	3	1.018	− 24	177.....	+49 35.42	1	+1.004	+0.015
148.....	34 57.60	3	1.023	− 20					
149.....	35 20.30	3	1.061	+ 16					
150.....	+35 40.00	4	+ 1.061	+0 015					

The normals are tabulated above in order of phase-time from conjunction in primary minimum. The variation of star c=B.D.+81°30 has been allowed for in the magnitude-differences $c-v$. The deviations from the computed curve are given under $O-C$.

Earlier observers, with few exceptions, have called attention to an asymmetry in the curve of primary minimum in the sense that the light increases more rapidly than it decreases. Following in Figure 1 the observations backward and forward from conjunction, one easily sees that the difference in brightness at corresponding phases increases steadily to the limits of the eclipse and persists beyond. The mean curves of other eclipsing variables, notably RV Ophiuchi, have presented similar characteristics. Effects of ellipticity and reflection are likewise apparent. It is readily seen that, when the minor oscillations of short duration are disregarded, it should be possible to represent the observed curve,

with the exception of the light-variation due to eclipse, by an expression of the form:

$$L = a + b \sin\theta - c \cos\theta + d \sin 2\theta - e \cos 2\theta,$$

in which the term $-c \cos\theta$ is the effect of reflection; $-e \cos 2\theta$ the usual symmetrical variation due to the ellipticity of figure; $b \sin\theta$ an effect due to the greater brightness of the advancing side

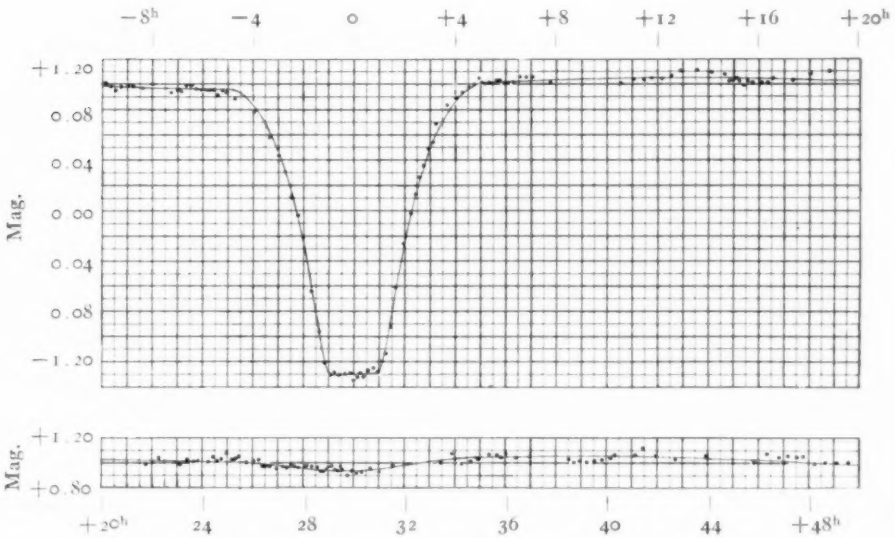

FIG. 1.—Showing normals of observation and computed curve. The curve is computed on the assumption of two-thirds' darkening at the limb and involves the effects of eclipse, ellipticity, reflection, brighter advancing side, and tidal lag.

of the bright star, or, if relevant, to increased brightness at periastron; and $d \sin 2\theta$ a term having maxima at $\theta = 45°$ and $\theta = 225°$. The least squares solution of all observations outside of minima resulted in the following set of values:

$$a = 0.9577 \pm 0.0031$$
$$b = 0.0078 \pm 0.0037$$
$$c = 0.0215 \pm 0.0047$$
$$d = 0.0214 \pm 0.0036$$
$$e = 0.0197 \pm 0.0052$$

The unit of intensity corresponds to the magnitude-difference $c-v=+1^{M}.064$, and θ is the longitude from conjunction in primary minimum. It will be seen both from the residuals $O-C$ and from the curve drawn out in Figure 1 that the observations outside of the minima are represented very satisfactorily.

Tidal lag.—The last two terms may be combined into

$$+0.0291 \sin (2\theta-42^{\circ}.6)$$

and considered as indicating that the long axes of the stars come into the plane of sight perpendicular to the orbit at $\theta=-23^{\circ}.7$ and again at $\theta=+156^{\circ}.3$. The disks present themselves to the observer in their maximum area at $\theta=+66^{\circ}.3$ and $\theta=+246^{\circ}.3$.

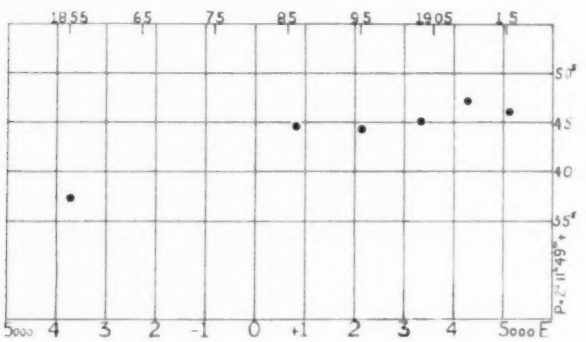

FIG. 2

This result suggested that an investigation of the period might furnish corroborative evidence of tidal evolution. A large number of minima of U Cephei have been observed since its recognition in 1880 as a variable. There is an apparently reliable observation by Schwerd not far from minimum on May 12, 1828, and a more doubtful one by Carrington on December 30, 1855. The mean periods, over the intervals between the large normals formed, given below and plotted in Figure 2, should be in error by less than a tenth of a second:

Epoch	Period $2^d11^h49^m+$
−3710	37.36
{−4049	30.45}
{−1685	44.72}
+ 805	44.59
+2133	44.29
+3340	45.05
+4273	47.15
+5126	46.00

If we divide the interval from 1828 to 1880 by including the Carrington observation, we obtain a much shorter period for the earlier and a much longer one for the later portion of this interval. These two values are bracketed in the second and third lines and do not appear in the plot.

A steady increase in the period amounting to about 9 seconds in 8800 periods, or in sixty years, combined with one or more small oscillations in its value is fairly apparent from Figure 2. Obviously this result depends largely upon the observation by Schwerd, which, however, there seems to be no reason for rejecting. It is very important that at least one minimum be well observed each year for a long time to come.

From the fact that a reflection effect on the darker star has been observed, the conclusion may be drawn that the rotation of the brighter star alone is more rapid than the revolution, with a resulting lag in its tidal elongation. On the plausible assumptions that the brighter star is twice as massive as the fainter star, that the mass of the whole system is twice that of the sun, and that the faint star rotates in its period of revolution, the daily angular velocity of the bright star has decreased 19′ in 60 years. The reader who feels that he has not before him entirely convincing evidence of tidal evolution in the system of U Cephei, will at least, I hope, fully realize the importance of the results we may hope to obtain from the study of the eclipsing variables, if not handicapped by discouraging lacunae in their observational history, and feel his responsibility in keeping them under observation.

Darkening at the limb.—The light curve of U Cephei, presenting as it does a total eclipse at primary minimum, and having been carefully observed throughout its entire extent, offered promising material for a determination of the degree of darkening at the limb. The formulae of solution when adapted to take care of the tidal lag become somewhat cumbersome, but may be simplified when, as in this case, the inclination of the orbit approximates close to 90°. It was found impossible to represent the secondary minimum either with a "uniform" or a "totally darkened" solution of primary minimum. Interpolating between these solutions it was found that the observed depth of secondary minimum indicated that the stars are approximately one-half as bright at the limb as at the center. In view of the observed variation of brightness on the sun's disk and the recent investigations in the theory of radiative equilibrium, it seemed probable that results of more general value would be secured from a new solution of primary minimum on the assumption that the brightness at the limb is one-third of the central brightness. This involved the calculation of tables for this degree of darkening, which will be published later if the promise of general usefulness is borne out. A good representation of primary minimum was effected and elements determined which gave very small residuals during secondary minimum. The explanation of the asymmetry of primary minimum, however, is still incomplete. The elements of the orbit and various results of interest follow:

Calculated duration of eclipse		10^h5^m6
Calculated duration of total or annular phases		1^h54^m8
Loss of light in primary minimum		2^M282
Loss of light in secondary minimum		0^M093
Maximum radius of larger star*	a_1	0.3225
Maximum radius of smaller star	a_2	0.2000
Minimum radius of larger star	b_1	0.3082
Minimum radius of smaller star	b_2	0.1911
Ratio of the radii of the stars	k	0.62
Ratio of axes of each star	$\frac{b_1}{a_1} = \frac{b_2}{a_2}$	0.956
Inclination of orbit plane	i	86°.4

* The unit of length is the radius of the relative orbit.

Eccentricity of orbit	e	0
Difference of light of sides of larger star	$2c$	0.043
Light of brighter side of smaller star†	L_2	0.8385
Light of brighter side of larger star	L_1	0.1615
Ratio of surface brightness:		
Of the bright sides of the two stars	$\frac{J_2}{J_1}$	13.5
Of the sides of the fainter star		1.36
Density of the smaller star	ρ_2	0.214
Density of the larger star	ρ_1	0.022
Magnitude of system at normal brightness (assumed)		6.8
Magnitude of bright star		6.9
Magnitude of faint side of faint star (as seen at primary minimum)		9.1
Tidal lag		23°.7

† The unit of light is the combined light of the brighter sides of the two stars at their maximum area and corresponds to $c - v = +1^{M}073$. The difference between advancing and following sides is disregarded throughout the table.

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THE SURFACE BRIGHTNESS OF THE GALACTIC SYSTEM AS SEEN FROM A DISTANT EXTERNAL POINT AND A COMPARISON WITH SPIRAL NEBULAE[1]

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ABSTRACT

Comparison of the galactic system with spiral nebulae as to surface brightness.—The results recently obtained by Kapteyn and van Rhijn for the distribution of stars of various magnitudes in our Galaxy have enabled the author to determine the surface-brightness of the galactic system, as viewed from a distant point in the direction of the galactic pole, for various distances from the center. The brightness of the central part is found to correspond to visual magnitude 23 per square second of arc, whereas probably all known spiral nebulae are brighter than this; in fact, many nebulae, including such well-known ones as that in Andromeda, are more than one hundred times as bright. This result indicates that spirals differ from the galactic system by being nebulous, or that they must be much larger or composed of more stars or of brighter stars. Our Galaxy is therefore far from being a typical spiral. Viewed edge on, the brightness of the Galaxy would be magnitude 21.2 per square second. These results are checked by comparison with the total amount of starlight in the direction of the pole and the Galaxy as computed from star counts.

Brightness of nebulous areas may be *determined photographically* by comparing the times of exposure necessary for barely perceptible images, with the exposure necessary to secure a trace of a large extra-focal image of a star of known magnitude. The constants for use with the 60-inch were determined in this way. Since in the case of extended nebulae, atmospheric disturbances and intermittency losses do not interfere with the continuity of exposure, images were obtained for a given magnitude per square second in the time required for images of stars about three magnitudes brighter.

In view of current discussion of the relation of spiral nebulae to the galactic system, it is of interest to compare the surface brightness of the spirals with that of our system as it would appear from a distant external point. This can be done with the aid of results recently obtained by Kapteyn and van Rhijn in an investigation of the density and luminosity laws.[2] The brightness at the center of figure of the system as seen in projection from the outside can also be computed from the numbers of stars in the successive intervals of apparent magnitude; but to determine the brightness at other points, the distribution of the stars in space

[1] *Contributions from the Mount Wilson Observatory*, No. 191.

[2] *Mt. Wilson Contr.*, No. 188; *Astrophysical Journal*, **52**, 23, 1920.

must be known, and this requires a knowledge of the laws of density and luminosity.

The density law gives the total number of stars per unit volume of space at various distances from the center of the system; the luminosity law specifies the number of stars of each unit of intrinsic brightness (absolute magnitude) in a unit volume of unit density. It is assumed that the *relative* frequencies of the absolute magnitudes are the same at all distances. For a constant volume, the total number of stars of each absolute magnitude therefore decreases, with increasing distance, in accordance with the density law.

In the form used here, these laws are a second approximation, based upon the data now available. Like the first, published by Kapteyn some years ago, they are mean results for all spectral types. Although still provisional, their precision is ample for the question considered.

I. INTEGRATION OF THE DENSITY-FUNCTION

Assuming the system to be a figure of revolution, symmetrical with respect to the galactic plane and with the sun at the center, Kapteyn and van Rhijn find for latitudes 0°, 30°, 60°, and 90° the stellar densities and corresponding distances given in Table I. The last column shows that in the direction of the pole the number of stars falls off rapidly, and at 1230 parsecs is only 0.01 that at the center of the system. In the plane of the Galaxy, on the other hand, a density of 0.01 occurs at a distance of nearly 9000 parsecs. The difference in these distances exhibits the well-known concentration of the stars toward the galactic plane. The authors estimate that the quantities in Table I are a very good approximation to distances of 1500 parsecs. In the direction of the pole this is practically the limit of the system, and it is clear that the light of the stars in the higher latitudes which are beyond this limit cannot be any important fraction of the whole.

Because of this circumstance, the external point from which the system is supposed to be observed is chosen in the direction of the galactic pole. Thus viewed, the system would be at least approximately circular in outline; and, if the point of observation were very distant, the stars would be unresolved and appear as a

luminous surface. It is the brightness of this surface that we wish to determine.

The numbers in Table I lead to the total density, D, in a direction perpendicular to the galactic plane at any distance from the center, while the luminosity law gives the total amount of light, l, in a volume of unit density. The combination of these results gives the quantity of light radiated from any point on the projected system in the direction of the distant external point.

As a preliminary, we must derive values of the density at intervals along the line of sight, i.e., along perpendiculars to the galactic plane, at various distances from the center. For the center of

TABLE I

STELLAR DENSITY AND DISTANCE FROM CENTER OF GALACTIC SYSTEM

Density	Galactic Latitude			
	0°	30°	60°	90°
1.00	0	0	0	0
0.40	910	320	250	250
0.16	1950*	710	490	450
0.063	3550	1320	800	660
0.025	5750	2140	1200	910
0.010	8910	3310	1700	1230
0.0040	13200	4900	2340	1580
0.0016	19100	7080	3090	2000

* The value by Kapteyn and van Rhijn, *loc. cit.*, Table VII, is 1450. Their results, however, are interpolated from Table VI of their article, which seems to require 1950, the distance used in the present discussion.

figure itself, the last column of Table I gives directly the required data. Selecting other points at the distances indicated in the heading of Table II, we find by graphical interpolation from the data of Table I the quantities in the body of Table II. Numbers in parentheses are densities in the galactic plane at distances from the center indicated in the heading. The other numbers are the distances from the plane, along the respective perpendiculars, at which occur the densities shown in the first column. Thus, at a point 400 parsecs from the center, the density in the plane is 0.78; and 400 parsecs perpendicularly above (and below) this point the density is 0.16.

To determine the total density D in the line of sight, consider a volume of square cross-section, 1 parsec × 1 parsec, extending from the center of the system to the distant point. From a plot of the numbers in the first two columns of Table II[1] it appears that for the first 100 parsecs from the plane the mean density is 0.87; for the second 100 parsecs it is 0.60; for the third, 0.41; etc. Hence the total density (multiplying by 2 to include stars on both sides of the plane) is

$$D = 200(0.87+0.60+0.41+0.27+\ldots\ldots) = 527.$$

The volume in question is therefore the equivalent of 527 cubic parsecs of unit density. The values of D for the remaining points were found in a similar manner; the results are in the lower part of Table II.

II. TOTAL LIGHT PER CUBIC PARSEC

If now l be the quantity of light, referred to a unit distance of one parsec, received by the observer from all the stars in a cubic parsec of unit density, the total light from one square parsec of the surface of the system seen in projection will be

$$L = Dl. \tag{1}$$

Since L is referred to unit distance, its equivalent absolute magnitude is

$$M = -2.5 \log L = -2.5 \log Dl. \tag{2}$$

We may calculate l approximately by reading from the luminosity-curve of Kapteyn and van Rhijn[2] the data in the second column of Table III, namely, the numbers of stars of each unit interval of absolute magnitude, M, in a volume of 1000 cubic parsecs of unit density. These numbers must be multiplied by the corresponding relative luminosities and summed.

Let the light of a star for which $M = 0$ be l_0; that for $M = -1$ will then be $2.512\, l_0$; for $M = -2$, $(2.512)^2 l_0$, etc. The logarithms

[1] Supplemented by a few values interpolated from Table VI of the investigation by Kapteyn and van Rhijn, *Mt. Wilson Contr.*, No. 188.

[2] *Loc. cit.*, Fig. 1.

TABLE II

DENSITIES ALONG PERPENDICULARS TO THE GALACTIC PLANE AT VARIOUS DISTANCES FROM CENTER

Density	Distance (ρ) in Parsecs										
	0	200	400	600	800	1000	1500	2000	3000	4000	5000
.....	(1.00)	(0.92)	(0.78)	(0.60)	(0.45)	(0.355)	(0.230)	(0.155)	(0.082)	(0.051)	(0.035)
0.40	250	210	170	120	50	.....	.....	.....	.....	.....	.....
0.16	450	430	400	350	320	270	150	.....	.....	.....	.....
0.063	660	690	700	700	680	660	585	500	400	.....	.....
0.025	910	950	1000	1050	1070	1090	1080	1020	1000	820	600
0.010	1230	1260	1320	1380	1450	1500	1590	1650	1800	1780	1750
0.0040	1580	1700	1780	1850	1920	2000	2200	2250	2400	2500	2600
0.0016	.....	.....	.....	.....	.....	2500	2700	2800	3100	3300	3450
D	527	490	428	360	306	276	216	183	143	114	93
M (1 sq. ps.)	−3.64	−3.57	−3.42	−3.23	−3.05	−2.94	−2.68	−2.50	−2.23	−1.98	−1.76
m (1 sq. sec)	22.93	23.00	23.15	23.34	23.52	23.63	23.89	24.07	24.34	24.59	24.81

of the coefficients of l_0 for the successive values of M are in the third column of Table III. Adding these to the quantities in the second column, we obtain the logarithms of the total light for each M, expressed in l_0 as a unit. The numbers themselves are in the last column. The total for all the stars is $55.5\ l_0$; hence for 1 cubic parsec $l = 0.0555\ l_0$.

TABLE III

LUMINOSITY-CURVE AND TOTAL LIGHT PER 1000 CUBIC PARSECS OF UNIT DENSITY

M*	Logarithm No. Stars 1000 Cubic Parsecs	Logarithm Relative Luminosity	Logarithm Total Light Each M	Total Light Each M
−11	4.4−10	4.4	8.8−10	0.1
10	5.3	4.0	9.3	0.2
9	6.2	3.6	9.8−10	0.6
8	7.0	3.2	0.2	1.6
7	7.7	2.8	0.5	3.2
6	8.3	2.4	0.7	5.0
5	8.8	2.0	0.8	6.3
4	9.3	1.6	0.9	7.9
3	9.7−10	1.2	0.9	7.9
2	0.1	0.8	0.9	7.9
− 1	0.4	0.4	0.8	6.3
0	0.6	0.0	0.6	4.0
+ 1	0.8	9.6−10	0.4	2.5
2	0.9	9.2	0.1	1.3
3	0.9	8.8	9.7−10	0.5
4	0.8	8.4	9.2	0.2
+ 5	0.6	8.0−10	8.6−10	0.0
			Total	55.5

* Unit of distance for M is 1 parsec.

The value of l can be more precisely obtained as follows: The luminosity law is very accurately expressed by

$$\left.\begin{aligned} \phi(M) &= A\frac{h}{\sqrt{\pi}}e^{-h^2(M-M_0)^2} \\ A = 0.0451,\quad M_0 &= +2.693,\quad h = 0.2218 \end{aligned}\right\} \qquad (3)$$

where $\phi(M)$ = number of stars of absolute magnitude M per cubic parsec of unit density. If l_M be the light of a star of magnitude M, then

$$\log\frac{l_M}{l_0} = -0.4\,M,\quad l_M = l_0 e^{-0.9211\,M}.$$

The total light of the stars in a cubic parsec of unit density is therefore

$$l = \int_{-\infty}^{+\infty} \phi(M) l_M dM = A l_0 \frac{h}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-h^2 (M - M_0)^2 + 11.60 M} dM,$$

which reduces to

$$\left.\begin{aligned} l &= A' l_0 \frac{h}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-h^2 (M - M_0')^2} dM \\ A' &= 0.0545, \quad M_0' = -3.11, \quad h = 0.2818. \end{aligned}\right\} \tag{4}$$

The integral is that of the ordinary error curve. With the coefficient $h/\sqrt{\pi}$ its value is unity; hence

$$l = 0.0545\, l_0 \tag{5}$$

in substantial agreement with the previous result. We adopt the value (5).

It will be noted that the number of stars for a given M is a maximum for $M = M_0 = 2.69$ (equation 3), while the maximum amount of light radiated by the stars of a given M is that corresponding to $M = M_0' = -3.11$ (equation 4). The latter result is checked, roughly at least, by the numbers in the last column of Table II, which indicate a maximum for $M = -3$. The integration of (3) between $-\infty$ and $+\infty$ with respect to M gives the result of Kapteyn and van Rhijn for the total number of stars of all magnitudes in a cubic parsec of unit density, namely, $A = 0.0451$.

III. SURFACE BRIGHTNESS OF GALACTIC SYSTEM. CONTROL OF RESULTS

From (5) it follows that

l = light of one star of absolute magnitude $+3.16$.

Equation (2) for the equivalent absolute magnitude of the light from one square parsec of the surface of the galactic system thus becomes

$$M = 3.16 - 2.5 \log D, \tag{6}$$

from which we find the values of M at the bottom of Table II.

For comparisons with spiral nebulae we need the apparent brightness of an area of known *angular* dimensions. Since we are dealing with a luminous surface, this is independent of the distance, for if the distance be doubled, the surface, and hence the amount of light, will be quadrupled; but the increase in light will be exactly compensated by the increase in distance, so that the amount received by the observer remains constant.

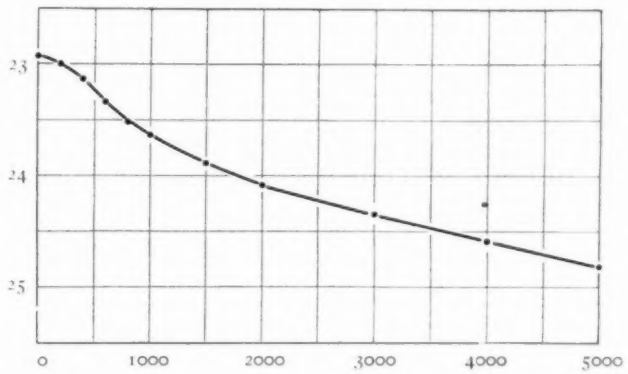

FIG. 1.—Surface brightness of the galactic system as seen from a distant external point in the direction of the galactic pole. Abscissae are distances in parsecs from the center of the system seen in projection; ordinates are the equivalent apparent magnitudes of the light received from an angular area of 1 square second of arc.

As the unit of angular area, we adopt one square second of arc. The required equivalent apparent magnitude is then

$$m = M + 5 \log 206265, \tag{7}$$

where M is the absolute magnitude of the light from one square parsec. The resulting values of m for various distances from the center of the projected system are given in the last line of Table II, and are illustrated in Figure 1.

Assuming the validity of the hypotheses upon which the density and luminosity laws are based—absence of absorption and invariancy of the luminosity law with distance—we may accept these results as reliable to distances of 2000 parsecs or so. Beyond this limit they may require some revision; but in any event it

seems clear that the magnitude curve is singularly flat, for in a radial distance of 2000 parsecs the decrease in surface brightness is only slightly in excess of a magnitude. This emphasizes anew the significance of the galactic plane as a structural feature of the system.

As a control upon these results, we may use the alternative method of procedure suggested in the opening paragraph. The surface brightness at the center of the galactic system as seen from an external point bears a simple relation to the quantity of starlight received by an observer on the earth, and this can easily be

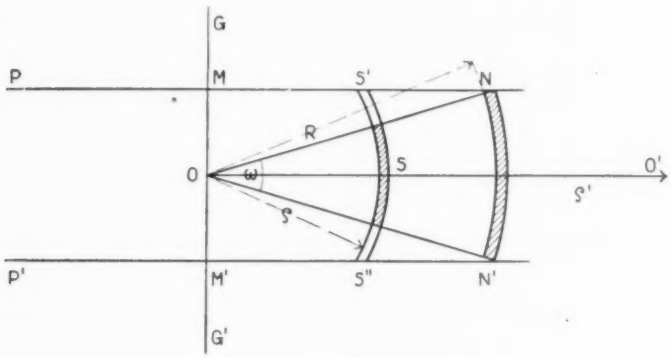

FIG. 2

calculated from the known numbers of stars in successive intervals of apparent magnitude.

Let GG' (Fig. 2) represent a small portion of the plane of the galaxy, O the center of the system, and OO' the direction of its pole. The distance, ρ', of the external observer at O' is assumed to be large as compared with the dimensions of the system whose limit is at NN'.

The light received by O' from the circular area NN' is that of all the stars in the cylinder $NPP'N'$. If the area of NN' be one square parsec, its equivalent magnitudes will be

$$M = 2.5 \log lD; \quad m = M + 5 \log \rho', \tag{8}$$

in which ρ' is the distance of O' in parsecs.

An observer at O, on the other hand, receives from an area of the sky defined by the solid angle ω the quantity of light, L', contributed by the stars in the cone NON'. To determine the relation of M and m to M' and m', the equivalent magnitudes of L', consider the thin shell S intercepted by the cone NON'. The volume of S is

$$dV = \omega\rho^2 d\rho,$$

ρ being the radius. If $\Delta(\rho)$ represent the stellar density at the distance ρ, the number of units of standard density in S will be

$$\Delta(\rho)dV = \omega\Delta(\rho)\rho^2 d\rho.$$

Since the light from all the stars in a cubic parsec of unit density, referred to unit distance, is l, the light at O from S is

$$dL' = \frac{l}{\rho^2}\Delta(\rho)dV = \omega l\Delta(\rho)d\rho. \tag{9}$$

This is independent of ρ, differences in distance being compensated by corresponding changes in the volume of the shell.

Thus a shell equal to and occupying the position of NN', and having the same density as S, will contribute the same amount of light as S. But since we are dealing with small values of ω, the shell $S'S''$ intercepted by the cylinder is sensibly equal to the shell NN'. Consequently the observer receives from S, the section intercepted by the cone, the same amount of light as he would receive from $S'S''$ if it occupied the position NN'. The total light at O from all the stars in the cone NON' is therefore the same as would be received from all the stars in the half-cylinder $NMM'N'$ if they were removed to the limit of the system NN' at the distance R.

The equivalent absolute magnitude of L' must therefore equal that of $L/2$, L, as before, being the light received at O' from the entire cylinder $NPP'N'$, reduced to unit distance. For a constant angular area a similar relation holds for the apparent magnitudes m and m'. The percentage error in these relations is of the order of $\sin^2 \frac{\alpha}{2}$, where α is the angular aperture of the cone NON', and is altogether negligible.

This result may be verified by integrating (9) to the limit of the system, $\rho = R$, whence

$$L' = \omega l \int_0^R \Delta(\rho) d\rho. \tag{10}$$

The value of the integral, with the approximation indicated above, is $D/2$, where D is the density integral already evaluated; and $\omega = A/R^2$, where $A = 1$ square parsec. Hence

$$m_0' = -2.5 \log L' = -2.5 \log lD + 0.75 + 5 \log R,$$

which is the equivalent apparent magnitude of the light in the cone referred to the distance R. We have, therefore,

$$\left.\begin{aligned} M' &= -2.5 \log lD + 0.75 \\ m' &= M' + 5 \log \rho' \end{aligned}\right\} \tag{11}$$

in which the apparent magnitude has now been referred to the same distance as that occurring in (8). From (8) and (11),

$$M' = M + 0.75, \quad m' = m + 0.75. \tag{12}$$

These relations hold for any direction in space, but it is presupposed that the line of sight passes through the center of the system. For galactic latitude 90°, the first column of Table II gives $m_{90} = 22.93$. This quantity and the value of m' derived from star counts at the galactic pole should satisfy the second relation.

The remaining values of m in Table II cannot be tested directly, but a strong presumption in favor of their reliability would be established should we find relation (12) also to be satisfied for latitude 0°. The density integral gives for the galactic plane $D = 2556$; the corresponding m is 21.22. From the density and luminosity laws we therefore find for a unit area of one square second, $m_0' = 21.97$ and $m_{90}' = 23.68$.

The most reliable star counts at present available are those of Kapteyn, reduced to the normal visual scale by van Rhijn.[1] From

[1] *Groningen Publication*, No. 27, Table V, p. 63, 1917.

these data we derive for latitudes 0° and 90° the numbers of stars for each interval of magnitude, multiply the results by the corresponding relative luminosities, and form the total in the manner followed in the case of the luminosity curve (Table III). The tables of van Rhijn extend only to $m=16$; but, since the fainter stars contribute only a small percentage of the total light received, it is possible to extrapolate the results to $m=19$ without introducing any serious error. The equivalent magnitudes for one square degree are thus found to be

$$m'_0=4.20 \qquad m'_{90}=5.98. \tag{13}$$

Using a different method of integration, van Rhijn finds 4.15 and 6.00, respectively.[1]

Reducing (13) to unit area and comparing with the results from the density and luminosity laws, we find the *equivalent apparent visual magnitudes of starlight for an area of one square second of arc:*

	Lat. 0°	Lat. 90°	
From density and luminosity laws	21.97	23.68	(14)
From star counts	21.98	23.76	
Difference	−0.01	−0.08	

Considering the uncertainties involved in the various numerical integrations, the agreement is excellent. It not only checks the calculations, in particular the results in Table II, but also shows that the revised density and luminosity laws found by Kapteyn and van Rhijn are in very good agreement with the best data now available.

IV. LIMITING MAGNITUDE OF INSTRUMENT FOR LUMINOUS SURFACES

We must now find values of the surface brightness for representatives of the spiral nebulae. A precise photometric determination is beset with troublesome difficulties; but approximations sufficient for the purpose can easily be obtained by noting the exposure time necessary to produce the first traces of a photographic image. We must know first, however, the limiting magnitude attainable in a given exposure with the instrument used.

[1] *Mt. Wilson Contr.*, No. 173, Table XI; *Astrophysical Journal*, **50**, 373, 1919.

For Seed 30 plates and an exposure of one minute with the 60-inch reflector, the limit for stars is from the fifteenth to the sixteenth magnitude, depending on the seeing. Results for point sources, however, cannot be used in the present instance without additional information which is not easily obtained. The limit has therefore been determined with the aid of luminous surfaces of known brightness, using for the purpose extra-focal images, from 2 to 6 mm in diameter, of stars of well-determined magnitude. The influence of seeing is then negligible, and the loss in photographic effect due to intermittent exposure, which always enters in the case of focal images, is avoided altogether, inasmuch as the sensitive film, except at the edge of the image, is constantly illuminated. On the other hand, there is possibly some disturbance arising from the phenomenon noted by Eberhard;[1] but for the very small photographic densities with which we are here concerned, the effect seems to be unimportant.

From several determinations made with plates of the emulsion used in deriving the limiting exposure for most of the nebulae we have:

Equivalent photographic magnitude of one square second of a luminous surface just shown by an exposure of one minute with the 60-inch reflector = 18.8 ± 0.3 (focal ratio 1:5). (15)

Neglecting differences in the absorption losses, this result applies to any telescope whose focal ratio is 1:5. Independent evidence that it is of the right order of magnitude will be given later.

Under good atmospheric conditions the difference in the limits for a point source and for a luminous surface of the area considered is therefore about three magnitudes. The value for surfaces is unexpectedly low and suggests that the intermittency loss is a factor of importance in determining the limiting magnitude for focal images of stars.

In the case of focal exposures to point sources, the optical image is much smaller than the photographic image, the diameter of the first dark ring of the diffraction pattern for the 60-inch reflector

[1] *Physikalische Zeitschrift*, **13**, 288, 1912.

being 0″.14. Owing to atmospheric disturbances, the optical image does not remain fixed in position, but oscillates rapidly, the amplitude of its excursions varying with the degree of the disturbance, so that no point within the boundary of the resulting photographic image is constantly illuminated. Suppose, for example, the area covered by the excursions to be 2″ in diameter—with good seeing, it is even smaller. If the light of a star of magnitude 15.8 were distributed uniformly over such an area, its equivalent magnitude per square second of arc would be 17.0, which is the *mean* brightness necessary to produce an appreciable photographic effect in one minute under the conditions of intermittent exposure. This is nearly two magnitudes above the limit corresponding to continuous exposure.

The difference, for the greater part, is apparently to be ascribed to intermittency loss. A subjective factor may also affect the result, for it is probable that smaller densities can be detected when the deposit covers several millimeters than when it is only 0.1 mm in diameter; this, however, will not affect the comparisons with nebulae whose images are large enough to be comparable with the extra-focal images of stars. In the case of very small nebulae, or of minute details in the larger objects, it may enter to some degree; and here, too, there may be an approach to the conditions that determine the limit for focal images of stars. Very small objects, therefore, may be somewhat brighter than the comparison with the limiting magnitude of luminous surfaces would indicate. Allowance for such an error would only strengthen the conclusion which is to be derived from the comparison of the surface brightness of nebulae with that of the galactic system.

V. SURFACE BRIGHTNESS OF CERTAIN SPIRAL NEBULAE

No great amount of data for the spirals is available at present, but an examination of a few plates of short exposure for such well-known objects as the Andromeda Nebula, N.G.C. 3254, 4254 (M 99), 4565, 4594, 4736, 4826 (M 64), and 5194 (M 51) leads to the conclusion that the surface brightness of the extended nebulosity near the nucleus is between the eighteenth and nineteenth photographic magnitudes. The details are as follows:

The Andromeda nebula.—From a 15^s exposure the sharply defined but irregular nucleus is estimated to be of the eleventh magnitude. This exposure shows distinct traces of attendant nebulosity. An exposure of 2^m reveals nebulosity covering an area about $2'$ in diameter.

N.G.C. 3254.—The nucleus, which is very diffuse, is discernible on a plate of 30^s exposure, and in 1^m is well shown, with attendant nebulosity. The inclination of the plane to the line of sight is probably 10° or 15°.

N.G.C. 4254 (*M 99*).—The diffuse nucleus appears in 30^s. The nebulous mass $2'$ *Nf* is well shown in 1^m; also one or two nebulous spots closer in. In 2^m the whorls are beginning to be outlined.

N.G.C. 4565 (*H V 24*).—The very diffuse nucleus is clearly shown in 1^m. An exposure of 2^m extends the nebulosity to a distance of $30''$. The nebula is seen edge-on.

N.G.C. 4594.—Also seen edge-on. The very diffuse nucleus is too bright on a plate of 1^m exposure for an estimate of its magnitude. This exposure shows nebulosity on both sides of the "absorption" band, extending, on the brighter side, to a distance of nearly $2'$ from the nucleus.

N.G.C. 4736.—From a 5^m exposure it is evident that 1^m would show nebulosity near the nucleus.

N.G.C. 4826 (*M 64*).—An exposure of 16^s shows the nucleus as a nebulous star; 32^s reveals nebulosity covering an area $10''$ in diameter. The ring $0'.9 \times 1'.5$ is easily traced on a plate of 64^s exposure.

N.G.C. 5194 (*M 51*).—Five minutes is sufficient to outline the arms throughout their length. The central part and the nebulous mass *Sp* are surprisingly strong. An exposure of 1^m would certainly show the brighter portions of the nebula.

These nebulae are among the largest and brightest of the spirals; but there are many others whose brighter parts seem to be comparable with those mentioned. Thus the photographs by Curtis, *Publications of the Lick Observatory*, **13**, Plates IV, V, and VI, especially Figures 38–69, suggest a striking similarity in the brightness of the more conspicuous portions of these objects. The reproductions are

from exposures ranging from approximately 1^h to 3^h. N.G.C. 4826, mentioned above, is among them (Fig. 54); and, aside from difference in size, is no more conspicuous than many of the others. The very small nebulae found in such numbers in the high galactic latitudes are doubtless fainter than the limit given above, although the difference is probably less than would be inferred from the long exposures necessary to record them; for many of them are so small that they must be compared with stars rather than with surfaces of appreciable extent.

Bearing in mind that the color-index of the central part of a spiral is of the order of 0.8 magnitude, it thus appears that the light of the brightest of these objects corresponds to an equivalent *visual* magnitude of from 17 to 18 per square second of arc; and that many of them are certainly brighter than the nineteenth visual magnitude.

VI. GENERAL REMARKS. CONTROL OF LIMITING MAGNITUDE FOR SURFACES

Certain questions are immediately raised by the values of the surface brightness given above, both for the spirals and for the galactic system. Thus for the Andromeda nebula we find a magnitude of 18, photographic, or possibly 17.5, visual, at a distance of $1'$ from the nucleus; and yet the object is easily visible to the unaided eye. The two results are not inconsistent, for it should be remembered that the value of the surface brightness refers to an area of 1 square second of arc.

The resolution limit of the eye may be placed at $2'$, which implies that the retinal image of a star has an effective angular diameter of this amount. Further, so far as the visual sensation is concerned, the result seems to be much the same whether the light of the source is concentrated in a point or whether it is distributed uniformly over a disk $2'$ in diameter. Referred to this unit of area, the equivalent visual magnitude of the central part of the Andromeda nebula is about 7.0. An adjacent zone $2'$ or $3'$ in width is not greatly inferior in brightness to the nebulosity near the nucleus, and, remembering the influence of the size of an object upon its visibility, it appears that the observed surface brightness

is quite compatible with the fact that the nebula is readily seen with the unaided eye.

Again, we easily see the star clouds of the Milky Way, a circumstance that at first seems irreconcilable with the results given in (14) for the equivalent brightness of star-light in galactic latitude 0°. Referred to an area 2′ in diameter this value becomes $m_0 = 12$. In other words, twelfth-magnitude stars distributed uniformly, at intervals of 2′, would appear as a luminous surface of approximately the same brightness as the average star-light in the Milky Way. The extent of the surface is here the determining factor. An isolated area 2′ in diameter of the brightness of the Milky Way could not be differentiated from the background of the sky, although a larger area of the same brightness would be visible. For stimuli near the threshold of visual sensation, the total energy of the stimulus is more important in fixing the limit of perception than the intensity per unit area of the retina.

Experiments by Russell[1] show that a fully rested eye, shielded from all extraneous light, is capable of perceiving luminous surfaces a degree in diameter when the equivalent brightness per 2′ area is as low as the sixteenth magnitude. In the open, under ordinary conditions of sky illumination, the limit is about 2.5 magnitudes brighter. In other words, the eye should distinguish from the average sky background a luminous area a degree in diameter whose brightness, in excess of the normal sky light, is of magnitude 13.5 per 2′ area. This is 1.5 magnitudes fainter than the calculated mean starlight in the Milky Way. The results above are therefore in no wise inconsistent with the ability of the eye to perceive faint sources of light. Moreover, the value in (14) is of the same order as that derived from the measures of the brightness of the sky by Yntema and by van Rhijn.[2]

Finally, since the estimates of the brightness of the spirals depend directly upon the limiting magnitude of surfaces that can be photographed in one minute, it is important that the value given in (15) be controlled. This has been done by means of photographs of the galactic clouds near Messier 8, placed at my

[1] *Astrophysical Journal*, **45**, 60, 1917.

[2] *Mt. Wilson Contr.*, No. 173, p. 18; *Astrophysical Journal*, **50**, 373, 1919.

disposal by Mr. Hubble. The exposures were made with an Ic Tessar of 6.5 inches focus, ratio 1:4.5, and a Voigtlander doublet of 16 inches focus, ratio 1:4. The limiting exposure that just outlines the clouds is 10^m for the former objective, and 7^m for the latter.

Since a 10-fold increase in the exposure corresponds to a gain of 2 in the limiting magnitude, we find in each case, starting from (15) and allowing for differences in the focal ratios, 21.0 as the photographic magnitude of the brightest parts of the clouds. To reduce to visual magnitude for comparison with the result from star counts, we must subtract the mean color-index. For this we have no exact data; but its value seems to be less than would be inferred from Fath's determination of the spectrum of the Milky Way.[1] Van Rhijn's recent investigation[2] of the brightness of the sky shows that the zodiacal light extends over the entire sky. Its intensity in the regions photographed by Fath seems to have been comparable with the starlight itself, and it is possible that the characteristic solar lines which appeared in his integrated spectrum may have been largely due to this source. Photographs by the method of exposure ratios, and direct determinations of color-indices by Shapley[3] show that there are many faint blue stars in the galactic clouds; and it seems unlikely that the mean color-index for these regions is in excess of 0.4 magnitude. This value is an estimate, but the error involved cannot be serious.

Another correction must also be taken into account, namely, the greater brightness of the sky background in the region photographed by Hubble as compared with that in the vicinity of the stars whose extra-focal images were used to derive the value in (15). This arises partly from difference in the zodiacal light and partly from a source of illumination, probably auroral in origin, whose intensity increases with the zenith distance. The influence of these factors, which has been determined with the aid of van Rhijn's

[1] *Mt. Wilson Contr.*, No. 63; *Astrophysical Journal*, **36**, 362, 1912. Slipher, *Publications of the American Astronomical Society*, 23d meeting, 1919, finds for the Milky Way a composite spectrum—solar type with abnormally strong hydrogen lines, which harmonizes with the suggestion presented here, namely, that the zodiacal light plays an important part in contributing the solar characteristics to the spectrum.

[2] *Mt. Wilson Contr.*, No. 173; *Astrophysical Journal*, **50**, 356, 1919.

[3] *Mt. Wilson Contr.*, No. 133; *Astrophysical Journal*, **46**, 64, 1917.

tables,[1] very nearly compensates the effect of the color-index, and we therefore adopt magnitude 21 for the visual brightness of the starlight in the galactic clouds which were photographed. The mean magnitude for latitude 0° found from star counts is 22. Counts for the particular region in question are not available, but from the fluctuations in the numbers of stars in other regions along the Milky Way it is clear that the excess of density in the vicinity of Messier 8 will account for most, if not all, of the remaining difference of one magnitude. This affords an independent check on the limiting brightness for luminous surfaces given in (15). The comparison is rough, but sufficient to confirm the previous result.

VII. COMPARISON OF NEBULAE WITH THE GALACTIC SYSTEM

In selecting nebulae for the comparison, preference has been given to those whose planes are nearly perpendicular to the line of sight in order that their magnitudes might be comparable with the brightness of the galactic system as seen from the distant external point on the axis of the system. The surface brightness of the latter at the center of figure we have found to be of the twenty-third magnitude (Table II).

The difference is therefore from 4 to 6 magnitudes. Had the comparison been with the nuclei of spirals, this difference would have been increased by two or more magnitudes; but leaving the nuclei out of consideration, we are led to the result that the surface brightness of many of the spirals is of the order of one hundred times that of the galactic system. The conclusions to be drawn from this result can be stated only as alternatives, and, unfortunately, settle nothing as to the constitution or size of the spirals.

If we suppose the spirals to consist wholly or mainly of extended nebulosity, the supposition in itself admits a structural dissimilarity, as compared with the galactic system, that may cover any sort of observed difference.

On the other hand, suppose the spirals to consist of collections of stars, isolated in space at such distances that they present more or less continuous, luminous surfaces. From a consideration of

[1] *Loc. cit.*

the quantities of light, l_S and l_G, received from unit angular areas of the surface of a spiral and of the galactic system, we then find

$$\frac{\delta_S}{\delta_G} = I \frac{T_G}{T_S}, \tag{16}$$

in which T_S and T_G are the thicknesses of the two systems measured along their polar axes, δ_S and δ_G, their mean stellar densities along the same axes, and I the ratio of intensities l_S/l_G.

Equation (16) is a relation between mean density, linear dimensions, and the intensity ratio I. If the dimensions of the spirals and the galactic system are of the same order,

$$\delta_S = I\delta_G, \tag{17}$$

in which for the brighter spirals I is of the order of 100.

If these objects are smaller than the galactic system, their stellar densities must be even greater, and exceed the value expressed by (17) in the ratio of T_G/T_S. Finally, if we suppose the densities to be the same,

$$T_S = IT_G, \tag{18}$$

which implies that the brighter spirals are very much larger than our system. The brighter spirals, at least, are therefore not directly comparable with the galactic system in the relation of stellar density to linear dimensions.

It is impossible to specify the percentage of the entire class of spirals to which this conclusion would apply, for we do not know at all accurately the total number within the reach of existing telescopes, and still less the number too faint to be thus revealed. We can say, however, that the value of I appearing in the formulae above is probably greater than unity for every known spiral. The value $I = 1$ would imply the existence of spirals with a surface brightness as low as the twenty-third magnitude, visual, or perhaps 23.8, photographic. An object of this brightness, if large enough to escape the photographic intermittency loss, would be just shown with the exposure time which faintly registers stars of the twenty-first photographic magnitude. With fast plates and good conditions this is approximately eight or ten hours, which is

near the maximum practicable limit. But the faintest spirals are apparently so small that the loss due to intermittency almost certainly comes into operation, with the result that the surface brightness of all these objects is probably greater than that of the galactic system; whence, for the known spirals, $I > 1$.

The alternative possibilities expressed by (17) and (18), which follow directly enough from the hypothesis of stellar constitution for the spirals, are therefore such that they place our own system at the end, if not actually outside, the series of known spirals when arranged according to one or the other of the two characteristics, density and linear dimensions. At the other end we find the extreme instances indicated by the formulae when we substitute for I a quantity of the order of 100.

MOUNT WILSON OBSERVATORY
July 1920

MAGNITUDES OF FAINT COMPARISON STARS FOR NOVA PERSEI, NO. 2[1]

By FREDERICK H. SEARES

ABSTRACT

Magnitudes of 36 faint comparison stars for Nova Persei No. 2 were determined by comparison with the Mount Wilson Standards at the North Pole.

Mount Wilson scale of photo-visual magnitudes compared with Barnard's gives values uniformly 0.8 magnitude larger for the range 12 to 15 magnitudes.

Brightness and color of Nova Persei No. 2 were determined on five dates during 1917–1918.

Photographic and photo-visual magnitudes of the faint comparison stars near Nova Persei, No. 2, which are included in the lists of Hagen,[2] Barnard,[3] and Aitken[4] have been determined by comparison with the Mount Wilson Standards at the North Pole.

TABLE I

OBSERVATIONS

Date	Plate		Exposure	H. A.	Seeing	Extinction
	Number	Brand				
1917 Sept. 22	4020....	Seed 27	2^m	50°E	3	$+0^M16$
22	4021....	Iso+C	3	50 E	3	+0.08
1917 Oct. 22	4125....	Iso+C	10	45 E	4	+0.09
22	4126....	Seed 27	8	46 E	4	+0.18
1918 Nov. 7	4778....	Iso+C	10	48 E	3	+0.08
7	4779....	Seed 27	5	38 E	3	+0.20
8	4789....	Seed 27	5	52 E	4	+0.15
8	4790....	Iso+C	10	42 E	4	+0.09
9	4802....	Iso+C	10	56 E	3	+0.06
9	4803....	Seed 30	2	48 E	3	+0.17

Five pairs of photographs were made with the 60-inch reflector as indicated in Table I. The Iso plates (Cramer, Instantaneous) were exposed behind yellow filter C. With the exception of those of the first pair, which received an additional exposure on the Pole,

[1] *Contributions from the Mount Wilson Observatory*, No. 192

[2] *Astrophysical Journal*, **13**, 233, 1901.

[3] *Astronomische Nachrichten*, **159**, 49, 1902; *Monthly Notices*, **80**, 710, 1920.

[4] *Lick Observatory Bulletin*, No. 8, 1901.

each plate was given three equal exposures in the order: Nova, Pole, Nova.

The photographs were measured by Miss Joyner and Miss Richmond in the usual manner. The scale-readings on the Polar Standards were plotted against their adopted magnitudes,[1] and

TABLE II

MAGNITUDES AND RESIDUALS

DESIGNATION			MEAN MAGNITUDES		COLOR-INDEX	RESIDUALS									
						Photographic					Photo-visual				
Hagen	Barnard	Aitken	Pg.	Pv.											
34	.....	.....	.....	10.63	.....	....	....	....	....	....	o	....	....	....	....
42	.....	.....	.....	10.75	.....	....	....	....	....	....	o	....	....	....	....
44	.....	.....	11.96	11.78	+0.18	o	....	....	....	....	+ 4	− 4	....	....	....
48	.....	.....	13.12	12.47	+0.65	+35	−24	−15	(+67)	+ 2	+18	−39	+ 4	+ 8	+11
49	15	.....	13.20	12.32	+0.88	−10	− 8	+20	+ 7	− 9	− 8	− 9	+29	− 4	− 8
54	.....	.....	13.24	12.69	+0.55	−10	− 4	+12	+ 7	− 5	+ 9	−13	+26	− 9	−12
56	14	*o*	13.37	12.64	+0.73	− 2	−24	+16	+24	−16	+ 1	− 7	+12	− 8	+ 2
57	.....	.....	16.22	15.50	+0.72	+22	+11	−28	+10	−13	....	− 9	....	+10	....
58	.....	.....	13.55	12.72	+0.83	− 8	−16	+14	+18	− 9	+ 4	−18	+20	− 6	+ 1
64	.....	.....	13.96	12.64	+1.32	+33	−28	− 4	−20	+17	− 4	−12	− 4	− 8	+30
65	.....	.....	14.23	12.80	+1.43	+21	−33	− 2	+ 7	+ 8	+ 7	−19	− 3	+ 2	+15
66	.....	.....	13.87	12.86	+1.01	+18	−27	− 7	+10	+ 8	− 8	−11	+ 3	+ 4	+13
68	.....	.....	13.61	12.82	+0.79	+24	−34	− 2	+17	− 3	o	− 8	+ 8	−12	+13
71	.....	.....	14.62	13.30	+1.32	−19	(−62)	− 1	+11	+ 8	− 4	−12	+ 8	− 3	+13
73	.....	.....	14.19	13.32	+0.87	−14	+ 5	− 3	+ 9	+ 4	− 7	+20	+ 4	−11	− 8
74	16	.....	14.07	13.20	+0.87	+40	−33	−18	+11	− 1	− 8	+12	− 2	− 9	+ 6
75	.....	.....	14.99	13.21	+1.78	+ 6	−29	+ 7	+14	o	− 7	−21	+11	+ 6	+10
76	.....	.....	14.70	13.36	+1.34	−13	(−54)	− 3	+16	− 1	− 6	− 1	− 2	+16	− 6
77	8	*i*	14.24	13.16	+1.08	+22	−26	−14	+22	− 5	− 6	−10	o	−17	+31
78	11	*l*	14.44	13.60	+0.84	− 7	− 6	+ 2	+22	−10	−11	+ 5	− 3	−15	+24
79	10	*k*	14.57	13.47	+1.10	− 9	−31	+12	+28	− 2	−10	+ 7	− 3	+ 8	+ 5
80	.....	.....	16.94	15.95	+0.99	....	+22	−24	− 1	+ 3	....	+14	....	+ 4	−19
81	2	*b*	15.57	14.53	+1.04	−35	− 3	+13	+12	+15	....	+19	− 9	− 6	− 4
.....	1	.....	16.44	15.19	+1.25	− 5	+22	−31	+16	− 1	....	+22	+ 3	− 4	−21
.....	3	*c*	14.42	13.67	+0.75	+ 2	−21	− 6	+13	+13	− 9	+10	− 5	−10	+16
.....	4	*e*	15.62	15.01	+0.61	−18	− 1	+ 5	+ 6	+ 7	....	+27	−19	+15	−23
.....	5	*f*	14.50	13.68	+0.82	− 9	−13	− 9	+28	+ 5	+ 1	+19	− 4	−15	− 1
.....	6	*g*	15.72	14.74	+0.98	− 8	− 4	− 7	+ 9	+10	....	+20	−19	+ 4	− 5
.....	7	*h*	15.20	14.38	+0.82	− 9	−22	+ 6	+14	+ 9	....	+ 7	− 3	+ 1	− 6
.....	9	*j*	15.61	13.94	+1.67	−14	− 4	− 2	+16	+ 6	+18	+ 4	+ 4	−23	− 1
.....	12	*m*	15.35	14.22	+1.13	−31	− 4	+17	+ 6	+14	(+55)	+14	−15	− 3	+ 2
.....	13	*n*	15.61	14.20	+1.41	−22	+15	+ 1	− 2	+ 6	+38	+16	−33	−12	− 7
.....	17	.....	14.71	13.82	+0.89	−16	−13	+ 1	+20	+ 9	+ 6	+14	− 8	−11	− 2
.....	18	.....	14.51	13.94	+0.57	− 8	− 8	+ 3	+ 8	+ 4	....	+23	−18	− 2	− 1
.....	20	*a*	16.86	15.88	+0.98	....	+18	−20	+ 5	− 2	....	+ 4	....	− 3	....
.....	.....	*d*	16.23	15.62	+0.61	−13	+15	− 1	− 2	o	....	(+44)	−13	+12	(−44)
			Systematic Difference			− 2	−10	− 2	+11	+ 2	+ 1	+ 2	− 1	− 3	+ 2

from the resulting curves were read the magnitudes of the comparison stars with the scale-reading as argument. The results were corrected for atmospheric extinction by the quantities in the last column of Table I, which reduce the magnitudes to the zenith.

The collected results are in Table II. The first three columns give the designation in the lists of Hagen, Barnard, and Aitken,

[1] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

respectively. The fourth and fifth columns contain the mean photographic and photo-visual magnitudes, and the sixth the color-indices. The remaining columns give the residuals for the separate plates in hundredths of a magnitude. Six of these exceed the rejection limit of 0.4 magnitude, and the corresponding magnitudes were accordingly omitted in forming the means; their residuals, referred to the adopted means, are inclosed in parentheses. The systematic differences at the bottom of the table are the deviations of the zero-points of the individual plates from the adopted mean zero-point.

TABLE III

COMPARISON OF BARNARD VISUAL WITH MOUNT WILSON PHOTO-VISUAL MAGNITUDES

Barnard Number	MW Pv. Magnitude	MW Pv. *minus* Barnard Visual	Corrected Difference
1	15.19	+0.69	$-0^{M}11$
2	14.53	.82	+0.02
3	13.67	.81	+0.01
4	15.01	(1.64)	(+0.84)
5	13.68	0.78	−0.02
6	14.74	1.11	+0.31
7	14.38	0.79	−0.01
8	13.16	.95	+0.15
9	13.94	.67	−0.13
10	13.47	.79	−0.01
11	13.60	.80	0.00
12	14.22	.78	−0.02
13	14.20	.70	−0.10
14	12.64	.69	−0.11
15	12.32	.77	−0.03
16	13.20	0.81	+0.01
17	13.82	1.00	+0.20
18	13.94	+0.80	0.00
Mean		+0.80	

The observations—of the photo-visual magnitudes at least—were undertaken at the request of Professor Barnard, as a basis for comparison with his visual measures, which he has kindly placed at my disposal. The large values of MW—Barnard (Table III) arise from the difference of 0.80 magnitude in the zero-points. Allowing for this, we find the differences given in the fourth column. These show no dependence upon magnitude, whence it follows that Professor Barnard's visual scale and the Mount Wilson photo-visual scales are parallel.

The color-indices of most of the stars lie between 0.7 and 1.2 magnitudes. Nevertheless, the range is sufficient to show that the relative color equation of the two series of measures is at least small. The photo-visual magnitudes are therefore directly comparable with the visual measures.

TABLE IV

BRIGHTNESS AND COLOR OF NOVA PERSEI, NO. 2

Date	Pg. Mag.	Pv. Mag.	Color-Index
1917 Sept. 22	13.79	13.28	$+0^{M}.51$
Oct. 22	14.40	13.55	0.85
1918 Nov. 7	13.44	13.16	0.28
8	13.81	13.39	0.42
9	13.45	12.78	+0.67

The differences for Nos. 4 and 6 are the only serious discordances, and suggest that the stars may be variable. In forming the zero-point difference used in Table III, No. 4 has been excluded, and No. 6 has been given reduced weight.

The brightness and color of the Nova derived from the five pairs of plates are as indicated in Table IV.

Only three of the stars in the Harvard list[1] fall within the limits of this investigation. A comparison with Wendell's results gives

Hagen No.	Wendell's Desig.	MW Pv.—Wendell	No. MW Obsns.
34	*c*	+0.06	1
42	*d*	−0.28	1
49	*f*	+0.09	5

[1] *Harvard Circular*, No. 66, 1902.

MOUNT WILSON OBSERVATORY
July 1920

EXPERIMENTS ON THE POSSIBLE INFLUENCE OF POTENTIAL DIFFERENCE ON THE RADIATION OF THE TUBE RESISTANCE FURNACE[1]

By ARTHUR S. KING

ABSTRACT

Radiation from the tube resistance furnace.—(1) *Effect of a small potential gradient.* To determine whether, as Hemsalech asserts, the currents through the ionized vapor, produced by a potential gradient of 1 or 2 volts per centimeter along the heating tube, were responsible for the emission of some of the furnace lines, a series of experiments was performed in which four different methods were used to reduce or eliminate the potential gradient acting on the vapor. In every case the relative intensity of the lines was found the same as with the higher potential gradient, for the same temperature. No evidence of the alleged low-voltage arc spectrum superposed upon the furnace spectrum was found. (2) *Origin of the radiation.* This result confirms the author's contention that whatever reactions between atoms or ions may cause the radiation, these reactions are caused and controlled by the high temperature alone, each temperature giving a definite spectrum. (3) The "*red fringe*" observed by Hemsalech is probably a chemical phenomenon, similar to a flame. It may be obtained when no potential difference is acting on the vapor.

When a substance is vaporized in a graphite tube by the passage of an electric current through the material of the tube or by any other method that will give sufficiently high temperature, it must be conceded that the high thermal condition may be accompanied by chemical actions and by effects resulting from the strongly ionized state known to exist near a hot surface. It has been maintained by the writer that, as the chemical and ionization conditions are caused by the high temperature and their intensities are controlled by it, the observed changes in the spectrum can properly be regarded as resulting from the variations of the temperature.

While, in the studies of metallic spectra thus far carried out, the furnace was usually operated according to a fixed procedure, many experiments have been made as to the effects of other methods of handling. Thus, with air present in the furnace chamber, the pressure has been varied from a few millimeters to over

[1] *Contributions from the Mount Wilson Observatory*, No. 193.

20 atmospheres. Carbon dioxide, oxygen, nitrogen, and hydrogen have also been used at pressures up to at least one atmosphere. The numerous changes in chemical action which must have resulted have failed to show anything that could be regarded as a real change in relative intensity of metallic lines when a given temperature was employed. The different distributions of vapor in an oxidizing atmosphere cause wider reversals of sensitive lines, and the general intensity of the spectrum, as measured by exposure times, may be greatly altered. Substances such as iron, which form carbides, show a more vigorous combination with the tube material when the furnace contains air at atmospheric pressure than when operated in a partial vacuum, and the intensity of the iron spectrum, as a whole, is increased. In all these cases, however, a certain temperature gives a definite set of lines, and, allowing for reversals, these lines show practically the same relative intensities.

When a potential difference is maintained between the ends of the tube, the presence of ionized vapor within the tube gives opportunity for the conduction of a certain fraction of the current by the metallic and carbon vapors. The potentials employed in my experiments, as measured by an alternating-current voltmeter, have ranged from less than 1 volt to about 2 volts per centimeter of the length of the tube. As the temperature is raised, the increased potential-drop, greater ionization, and higher resistance of the graphite tube will combine to give more current through the vapor. The possibility of such currents through the vapor is thus evident. Whether, under the usual conditions of operating the furnace, these currents have any effect in exciting the emission of lines not produced at the same temperature when there is no current in the tube is another matter, and it is for the purpose of testing this question that the following experiments were undertaken.

In his recent papers[1] Dr. G. A. Hemsalech asserts that a large class of lines requires, before they are emitted, the presence in the furnace tube of a low-tension arc, such as would result from the passage of a considerable current through the vapor. These are

[1] *Philosophical Magazine*, **36**, 209, 281, 1918; **39**, 241, 1920.

lines which I have found to be faint or absent at furnace temperatures which produce another large class, the low-temperature lines; but the lines first named rapidly gain intensity at higher temperatures. The question is whether at these higher temperatures the conductivity of the vapor in the tube has become so great that the radiation of a low-tension arc is superposed on that of the furnace. In a paper[1] published after the appearance of the second article by Dr. Hemsalech, I showed that the lines which he had not observed until his furnace reached a temperature of about 2500° C. were obtainable with temperatures as low as 1800° and were distinct at 2000°, for which Dr. Hemsalech did not suspect a disturbing conductivity of the vapor.

In his last paper, Dr. Hemsalech attempts to account for the discordance between his results and mine on the ground that the potential-drop per centimeter which I used to produce a temperature of 1800° was about the same as that for which he obtained 2500°. This he attributes to my use of an unprotected tube in a vacuum chamber and of an alternating voltage which has high momentary values, whereas he has used a jacketed tube with direct current. Further, by passing a current through a thin graphite plate, he found that, when this was heated to a high temperature, a red glow appeared on the under side of the plate. This glow was sharply defined and was found to be deflected by a magnetic field in a direction which indicated that a part of the heating current passed through the vapor adjacent to the plate. The "red fringe" thus produced gave a spectrum consisting of carbon flutings and lines of titanium and vanadium, elements regularly found in the Acheson graphite. These flutings and metallic lines showed a sharp falling off in intensity at the limit of the "red fringe," and this fact, together with the deflection of the fringe by a magnetic field, led Dr. Hemsalech to consider that when they occur in a tube furnace (which can be regarded as a rolled-up plate), they are due to a passage of the current through the vapor within the tube.

These two points, the relatively large potential-drop which I used in producing the test lines of iron at low temperature, and the apparent electrical nature of the "red fringe," are the outstanding

[1] *Mt. Wilson Contr.*, No. 162; *Astrophysical Journal*, **49**, 48, 1919.

features of Dr. Hemsalech's argument that the tube furnace is to some extent a low-tension arc.

The "red fringe," the color of which is due to a band spectrum, is, I believe, mainly a chemical phenomenon which I shall show later occurs near heated carbon when no current is being carried by it. The sharp boundary of the fringe is difficult to account for on electrical grounds, but would seem to be readily explained by the sudden cessation of a chemical reaction, corresponding to the production of the sharply defined envelopes of flames. The well-known deflection of ionized vapors in flames would lead one to expect the effect of the magnetic field which Dr. Hemsalech observed. The evidence from the direction of the deflection that the ions were carrying part of the heating current would have been more complete if the current had been reversed, while the direction of the field remained the same. The temperature of the fringe vapor must have been much below the measured temperature of the graphite plate, on account of the strong gradient inevitable at the exposed surface. The "red fringe" is, I believe, the same as the red glow which appears regularly in the tube furnace at temperatures well below 2000° C. when a fresh tube is first heated. It fades after a short time, with the temperature unchanged. I have always ascribed it to a transient condition while some impurities of the graphite are being burned out.

That certain lines appear in a mass of ionized vapor through which a fraction of the heating current may be passing, clearly does not justify the conclusion that these lines require the current for their production. Experiments are needed in which, with the vapor heated to about the same temperature, the possibility of a current is reduced or eliminated. Such experiments, which I have carried out by four different methods, will now be described. Three of these methods involved different arrangements of the tube furnace, while in the fourth the tube was heated by an external arc.

1. *The use of direct current with protected tube, and comparison of direct and alternating currents with unprotected tube.*—A large direct-current generator, just installed, made it possible to use with the furnace heavy currents which have been available hitherto only with transformer voltage. In order to retain the heat and thus

permit the use of lower potentials, the tube was jacketed with a protecting tube of graphite and outside of this with cakes of partially fused carborundum. In the earlier experiments with this furnace,[1] a protecting jacket of this kind was used. It has the disadvantage that the temperature gradually rises during a considerable interval while the jacket is being heated, and with the alternating current, no method is available for changing the voltage by small steps to counteract this increase. For the regular examination of spectra at different temperatures, where constancy during the exposure is a prime requisite, the tube was left unprotected inside the vacuum chamber. This arrangement, as Dr. Hemsalech has noted, requires relatively high potentials to give the desired temperatures, and the peak voltage during each alternation produces momentary values of the electric field higher than those given by direct current for the same tube temperature. The possible effect of this was easily tested by using a protected tube, with current from the new generator, the adjustment of whose field permits a fine variation of the voltage to allow for heating of the jacket. Temperature measurements were made at the center of the inner wall of the tube by means of a Leeds and Northrup pyrometer of the Morse type, whose readings yield the same temperatures as the less convenient Wanner pyrometer previously used. The potential-drop was measured across the electrode pipes outside the furnace chamber and included not only that over the tube but also across six contacts within the chamber, whose resistance was not negligible.

Photographs of the iron spectrum were made with the furnace at 1950° and 1800° C. For the latter temperature the voltage, after the jacket was fully heated, remained steady at 12 volts, or 0.6 volt per cm of the tube, neglecting the contact resistances. Allowing for the probable drop over these, it is likely that not more than 10 volts, or 0.5 volt per cm, were involved in the production of the spectrum observed. In this spectrum, all of the strong iron lines of the λ 4900 group were distinctly though faintly registered. These are the test lines which Dr. Hemsalech maintains do not appear in the tube furnace with direct current below 2500°, at

[1] *Mt. Wilson Contr.*, Nos. 28, 32; *Astrophysical Journal*, **28**, 300, 389, 1908.

which temperature the potential-drop per cm in his experiments was about twice that used here. The rest of the spectrum at 1800° was the regular low-temperature spectrum, with the lines of Classes I and II strong and those of Class III, to which the group near λ 4900 belongs, beginning to develop. For instance, the strong arc line λ 4260 of Class III was less than one-fifth as strong as the adjacent λ 4258 of Class IA, a relation which can be reversed at higher temperatures. The prominent impurity lines of titanium and vanadium were strong, and the "cyanogen" band at λ 3883 was well developed. It has been shown in previous experiments that this band and those of the carbon spectrum are much stronger in air at atmospheric pressure than in the partial vacuum used in these experiments. The spectrum at 1950° differed from that at 1800° chiefly in the rapid strengthening of the lines of Class III, λ 4260 now being only slightly weaker than λ 4258.

Experiments were next made with the usual unprotected tube, charged with iron filings, a photograph being taken at a certain temperature with alternating current, followed by one with direct current adjusted to give the same pyrometer reading. Pairs of spectrograms were thus made for temperatures of 1750°, 1800°, and 2200°. Slightly higher voltages were of course required than in the case of the protected tube. The spectra at a given temperature were practically identical, the same lines being present, with the same relative intensities, whether alternating or direct current was used.

These experiments serve to establish the emission of lines of Class III, such as those of iron near λ 4900, with a very low potential-drop given by direct current, and at the temperatures at which I have regularly observed them with the furnace excited in the usual manner. That Dr. Hemsalech did not observe these lines in his tube furnace resulted, I believe, simply from the faintness which must be expected in the case of lines of Class III. Visual observations cannot be expected to show them, especially with low dispersion, until they have attained the brightness associated with a higher temperature. They may also be lost on a photographic plate having decided contrast, unless it be given long exposure, though the low-temperature lines may be quite distinct.

With a softer plate, and exposure sufficient to produce a strong general intensity, however, the lines of Class III appear distinctly without over-exposure of the rest of the spectrum.

2. *The furnace with insulated inner tube.*—This method and the one following were employed for the purpose of reducing or excluding any action on the vapor by the potential-drop usually present between the ends of the furnace tube. A double tube was arranged as in Figure 1, consisting of a tube of 6.25-mm bore, inside of the regular 12.5-mm tube which carried the current. The inner tube was turned down to give a clearance of about 1 mm between it and the outer tube, and was insulated from the latter by short pieces of quartz tubing at the ends, which were kept cool by the contact blocks. The inner tube, which contained the iron vapor, should have conducted much of the current which otherwise would have passed through the vapor, thus largely avoiding the conditions of a

FIG. 1.—Arrangement of insulated inner tube

low-tension arc. The ammeter gave no indication of an increase of current resulting from the low-resistance inner tube, and when alternating current was used on the tube at temperatures of 1700° and 2000° C. the spectra were quite the same as for the single tube at the same temperatures. The experiments were carried out both in a partial vacuum and at atmospheric pressure.

3. *Spectrum photographed with no current in tube.*—The furnace tube, when well protected by an inclosing jacket, cools so slowly that its spectrum fades only gradually when the current is broken. This persistence of the stronger low-temperature lines, which frequently remain visible for several minutes, has often been noted. The photography of this spectrum after the current had ceased was now undertaken.

A jacket was used about the tube similar to that described in the experiments with direct current, and the furnace was excited to a temperature of 2350° C., the window toward the spectograph being covered by a shutter. The transformer switch was opened and

the same hand was then used to open the shutter, thus insuring an interval of at least half a second after the current was broken before the exposure was started. In fifteen seconds the temperature fell below 1500°; after which the tube was heated as before and again exposed with no current. A total exposure of 30 minutes by this method served to build up a spectrogram of sufficient strength. Without opening the furnace, the spectrum was then photographed with a continuous run at reduced voltage, giving a temperature of about 2100° C. The two spectra were the same with respect to lines present and relative intensities; the test iron lines of Class III were distinct, as were also the lines of titanium and vanadium and the cyanogen bands.

This experiment would seem conclusive as to the possibility of producing, without a potential difference, the lines which Dr. Hemsalech considers require the presence of a low-tension arc. Dr. Hemsalech, has, however, also observed a persistence of the group at λ 4900 after the current had been broken and has compared it with the residual luminescence which he observed in spark spectra[1] when the metallic vapor was blown out of the electric field by an air blast. It seems scarcely permissible to compare this phenomenon of the spark with the present experiment, as the time intervals are of quite a different order. The duration of visibility of iron lines in the spark vapor was about 0.0002 sec. Even a line so easily produced as λ 4227 of calcium lasted but 0.0003 sec. The reader must judge whether the radiation of iron lines of Class III and other test lines during several seconds can be reasonably ascribed to the better protection of the vapor afforded by the furnace tube.

4. *Furnace tube heated by an external arc.*—In this experiment, the furnace tube was in effect a horizontal crucible in which iron was vaporized by a heavy-current arc between horizontal carbon electrodes which were brought together beneath the tube. The arrangement is shown in Figure 2. A is a short graphite tube of 6.25-mm bore and 12.5-mm outside diameter. An alternating-current arc carrying 200–300 amperes was formed between the carbon rods B. These were insulated by carborundum plates C

[1] *Comptes Rendus*, **150**, 1743, 1910; **151**, 220, 1910.

from the graphite blocks D which inclosed the tube and the arc. The ends of the furnace tube A were fitted into sections of graphite tube of 19-mm outside diameter which passed through holes in the graphite housing and permitted a view of the interior of the tube. A screen at the end toward the spectrograph excluded from the slit all light except that from the vapor within the tube. After each run the tube was carefully examined to see that no hole had been formed which would permit contamination by vapor from the arc.

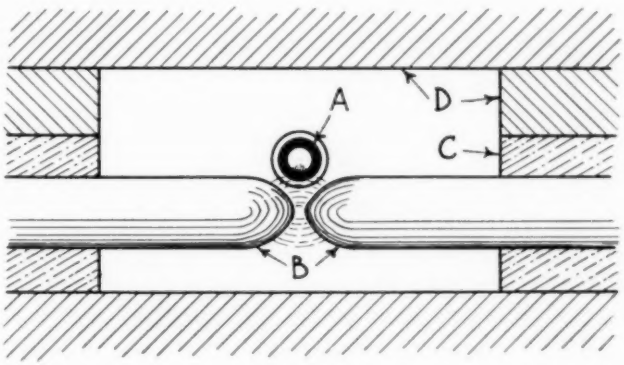

FIG. 2.—Furnace tube (A) heated by external arc (B)

This furnace is not to be recommended for regular work as a substitute for the tube heated by a current, for the shifting of the arc about the ends of the electrodes caused much variation in the heating of the tube. At intervals, however, the tube became hot enough to emit a rich spectrum, which was photographed in the first order of a 1-meter concave grating. The spectra thus obtained showed no essential difference from the numerous spectrograms made with the regular tube furnace at about 2000°. The test lines of interest in this investigation, consisting of the iron lines of Class III near λ 4900, the titanium and vanadium lines given by impurities in the graphite, and the "cyanogen" flutings were all present. The latter were of the strength usual with the tube resistance furnace at atmospheric pressure, and the relative intensities of the metallic lines showed no differences from those given by the resistance furnace.

An interesting feature was the production in the tube of the "red fringe" noted by Dr. Hemsalech near his graphite plate heated by a current. This red glow appeared at the bottom of the tube, next the arc, and occasionally, for short intervals, filled the tube; but usually it extended across about half the diameter, with a sharp boundary above. Above the red glow a yellowish vapor often appeared. The grating photographs, which in the first order show incomplete astigmatism, correspond closely with those taken by Dr. Hemsalech near the hot plate. The low-temperature lines of iron and such impurity lines as λ 4227 of calcium and λ 4607 of strontium, maintain nearly the same intensity across the diameter of the tube; while the iron lines of Class III, the titanium and vanadium lines, and the cyanogen bands are strongest in the portion occupied by the red glow, fading rapidly at its limit. The whole effect seems to me to indicate a chemical condition adjacent to the heated carbon, regulated by the temperature and without any dependence on the passage of a current through the vapor.

It seems a fair conclusion from these experiments that the temperature classification used by the writer for furnace spectra is a sound one, and that any conduction of current by the vapor which may take place at the temperatures on which the classification depends is quite without effect in modifying the spectrum. Instead of accepting Dr. Hemsalech's view that the low-temperature lines are due to a thermo-chemical excitation, and that the high-temperature lines, including those of Class III, require a passage of the current equivalent to a low-tension arc, I believe the experimental evidence indicates that the rôle of chemical action is uncertain, but that it probably acts mainly by affecting the general intensity; and that the effect of such current as may be conducted by the vapor at moderate furnace temperatures is negligible. I have laid stress, as does Dr. Hemsalech, on the lines of Class III, since these are the chief indicators of changing temperature in the furnace; and the object of this investigation has been to show that these lines, which, with the low-temperature lines of Classes I and II, make up the bulk of the furnace spectrum, do not require the passage of a current through the vapor. The lines of Classes IV and V, which are absent or faint at high furnace temperatures,

include several varieties between which the furnace does not distinguish. Such lines may be of the diffuse type for which the electrical sources are especially favorable; they may partake of the nature of enhanced lines; or they may be lost in the furnace spectrum through extreme faintness. The question is for the present left open as to how far the effects of higher furnace temperatures, say above 2400° C., may be complicated by conduction of current. For present purposes this is not vital, but would become important if the temperature-range were transferred to a higher level, such as would be needed for the spectrum of tungsten, this metal melting at about 2700° C. when in contact with carbon. An indication is given, however, of the weakness of electrical excitation even at the higher furnace temperatures by the fact that of the enhanced lines of titanium, given in the arc of about the same intensity as the stronger arc lines, only a few are shown as traces in the furnace at 2600° C. The question remains in this case as to whether the high temperature begins to excite these radiations, or whether a weak electrical action causes their emission.

SUMMARY

The material in this paper may be summed up by saying that we have no evidence that so small a potential-drop per centimeter as that existing in the tube furnace can excite the test lines and bands observed, and that, for the temperatures required to classify the lines of most metallic spectra, the experiments here reported show a definite spectrum for a given temperature when the potential acting on the vapor is reduced, or as far as possible eliminated.

The writer was assisted in this work by Mr. Sinclair Smith, especially in the handling of the arc-heated furnace.

MOUNT WILSON OBSERVATORY
July 1920

RADIAL VELOCITIES OF EIGHTEEN STARS

By ADOLF HNATEK

ABSTRACT

Radial velocities of ten binaries (12, 15 κ, 42, 50 Cassiopeiae; 34 ψ^2, 68, 71, +62°1637 Draconis; +65°369 Camelopard.; 21 η Ursae Minoris) *and of eight other stars* (16, 19, +59°2395, +83°104 Cephei; 73, +73°835 Draconis, 25 θ Ursae Majoris, 23 δ Ursae Minoris), determined from 45 spectrograms taken in 1913–1917, are given in Tables I–II.

The radial velocities given in Table I have been obtained with the Coudé spectrograph of the University Observatory, Vienna.

From my earlier determinations of radial velocity, in *Astronomische Nachrichten*, **197,** 185, 1914, the values for the stars 12 Cassiopeiae and 15 κ Cassiopeiae seemed to be not in sufficient accordance with the values given by Adams (*Astrophysical Journal*, **35,** 163, 1921), by Lee (*ibid.*, **39,** 39, 1914), and in *Lick Observatory Bulletin* No. 199. An investigation of the plates showed a blending of lines caused by a very granular appearance of the emulsion, probably due to different temperatures of the baths. In that case, a cementing of the plates by Canada balsam to a cover-glass entirely removes this structure, and the spectra become clear and the lines distinct. In this way I have prepared and remeasured a few plates, and have secured the new values given in Table II, which are to be preferred to those published in *Astronomische Nachrichten*, **197,** 185, 1914.

The stars 21 η Ursae Minoris, 34 ψ^2 Draconis, +62°1637 Draconis, 68 Draconis, 71 Draconis, and 42 Cassiopeiae are spectroscopic binaries.

The binary character of the stars 15 κ Cassiopeiae, 50 Cassiopeiae, +65°369 Camelopardalis and 12 Cassiopeiae, which were previously announced as binaries in *Lick Observatory Bulletin* No. 199 and this *Journal*, **35,** 163, 1912, is here confirmed.

TABLE I

Star, Mag., and Type	No. of Plate	Date	G.M.T.	Exp.	Radial Velocity	No. of Lines
					km	
25 θ Ursae Majoris	B688.....	1915 June 16	8^h53^m	60^m	+ 1.8	5
5.1, K5	B903.....	1916 June 25	8 36	60	+ 0.8	10
				Mean	+ 1.3	
21 η Ursae Minoris	B667.....	1915 June 5	10 17	30	+35.7	9
5.2, F	B680.....	1915 June 11	8 58	40	−20.9	13
	B904.....	1916 June 25	9 25	30	+17.8::	6
23 δ Ursae Minoris	B700.....	1915 July 5	9 56	20	−12.5	8
4.7, ?	B705*....	1915 July 12	8 58	50	− 9.2	6
	B713.....	1915 July 19	9 43	30	−16.8	7
				Mean	−12.8	
34 ψ^2 Draconis	B717†....	1915 July 20	10 9	50	+ 8.5	13
5.7, F	B725†....	1915 July 26	10 5	50	− 8.1	15
	B907.....	1916 July 31	8 18	56	+20.4	6
+62°1637 Draconis	B732.....	1915 Aug. 2	8 33	60	+21.6	8
6.0, A	B741.....	1915 Aug. 23	8 37	70	+14.9	6
+73°835 Draconis	B744‡....	1915 Aug. 24	8 10	70	−37.4	5
5.4, K?	B757‡....	1915 Aug. 31	8 5	70	−38.9	6
	B910‡....	1916 Aug 8	9 3	40	−40.9	9
				Mean	−39.1	
68 Draconis	B778.....	1915 Sept. 21	7 47	70	−16.3	14
5.9, ?	B912.....	1916 Aug. 25	7 33	60	− 9.5	10
71 Draconis	B782.....	1915 Sept. 22	7 28	70	−23.3	12
5.9, A	B915.....	1916 Sept. 9	7 14	62	+27.5	9
73 Draconis	B564§....	1914 Sept. 16	8 4	50	−12.6	19
5.4, A2p	B916.....	1916 Sept. 9	8 6	35	−13.5	11
	B923.....	1916 Sept. 17	7 26	35	−17.5	12
				Mean	−13.4	
+65°369 Camelopard.	B342.....	1914 Jan. 5	8 32	30	+ 7.9	5
4.5, M						
+83°104 Cephei	B1125....	1917 Jan. 22	7 26	50	+32.6	11
5.9, B3						
+59°2395 Cephei	B789.....	1915 Oct. 12	8 15	50	+ 4.1	5
5.7, A	B953.....	1916 Sept. 25	6 30	50	+ 5.9	6
				Mean	+ 5.0	

* Spectrum very faint.

† A few lines of Plate B717 and B907 give negative instead of positive values. Probably double lines are present.

‡ The spectra are very faint. § A very fine spectrum, with good and distinct lines.

|| An excessively narrow spectrum. Plate B381 confirms the sign of radial velocity, but is too poor for measurement. According to *Lick Observatory Bulletin* No. 199. the radial velocity varies.

TABLE II

Star, Mag., and Type	No. of Plate	Date	G.M.T.	Exp.	Radial Velocity	No of Lines
					km	
12 Cassiopeiae	B252.....	1913 Oct. 30	9^h24^m	50^m	+30.7	7
5.6, B9	B255.....	1913 Oct. 31	8 52	60	+29.0	9
	B280.....	1913 Nov. 21	8 23	76	+27.7	9
15 κ Cassiopeiae	B231.....	1913 Oct. 13	8 3	42	−19.8	10
4.4, B	B244.....	1913 Oct. 25	10 25	18	−20.5	11
	B248.....	1913 Oct. 27	9 5	18	−29.1	14
	B277.....	1913 Nov. 19	9 37	30	−21.5	7
42 Cassiopeiae	B258.....	1913 Nov. 1	10 16	46	+25.8	19
5.4, A	B1109*...	1916 Dec. 28	9 46	26	+12.8	8
50 Cassiopeiae	B245.....	1013 Oct. 25	10 46	18	−13.6	7
4.2, A	B256.....	1913 Oct. 31	9 44	16	−19.2	10
	B259.....	1913 Nov. 1	10 51	16	−18.4	10
16 Cephei	B236.....	1913 Oct. 14	9 18	40	−28.5	19
5.3, F	B278	1913 Nov. 20	7 28	64	−29.3	13
	B567.....	1914 Sept. 26	7 29	60	−27.1	8
				Mean	−28.3	
19 Cephei	B232.....	1913 Oct. 13	8 58	62	−26.2	10
5.4, Oe5	B238.....	1913 Oct. 18	7 35	50	−24.6	5
	B246.....	1913 Oct. 27	8 7	60	−24.7	7
				Mean	−25.2	

* Spectrum very faint.

UNIVERSITY OBSERVATORY
VIENNA
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